

GEOMETRY

Right Triangles:

Pythagorean Theorem	Proportions in a Right Δ	Special Right Δ s												
$a^2 + b^2 = c^2$ Famous Pythagorean triples: (3, 4, 5); (5, 12, 13); (9, 12, 15); (6, 8, 10); (8, 15, 17)	Altitude drawn to the hypotenuse. $alt.^2 = seg \cdot seg$ $h^2 = xy$ $leg^2 = hyp \cdot seg$ $b^2 = cx$	30-60-90: <table style="display: inline-table; border-collapse: collapse; margin-left: 20px;"> <tr><td style="border-right: 1px solid black; padding: 2px 5px;">30</td><td style="border-right: 1px solid black; padding: 2px 5px;">60</td><td style="padding: 2px 5px;">90</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 5px;">1</td><td style="border-right: 1px solid black; padding: 2px 5px;">$\sqrt{3}$</td><td style="padding: 2px 5px;">2</td></tr> </table> 45-45-90: <table style="display: inline-table; border-collapse: collapse; margin-left: 20px;"> <tr><td style="border-right: 1px solid black; padding: 2px 5px;">45</td><td style="border-right: 1px solid black; padding: 2px 5px;">45</td><td style="padding: 2px 5px;">90</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 5px;">1</td><td style="border-right: 1px solid black; padding: 2px 5px;">1</td><td style="padding: 2px 5px;">$\sqrt{2}$</td></tr> </table>	30	60	90	1	$\sqrt{3}$	2	45	45	90	1	1	$\sqrt{2}$
30	60	90												
1	$\sqrt{3}$	2												
45	45	90												
1	1	$\sqrt{2}$												

ALGEBRAIC TECHNIQUES

1. Complex Fractions:

- a. Find LCD of all fractions;
- b. Determine what each fraction is missing to have this LCD as its denominator;
- c. All "smaller" denominators will be canceled out.

2. Solving Equations:

Absolute Value	a. Isolate $ $ to one side of the equation; b. Break $ $ into $+$ and $-$ to make two equations and solve each equation; c. Check the solutions and reject any derived roots.
Fractional	a. Find LCD and determine what each fraction is missing to have this LCD; b. Cancel out the common denominators and solve equation that remains; c. Check solution and reject any roots that makes original fractions undefined.
Radical	a. Isolate radical sign to one side of the equation; b. Square both sides and solve the equation; c. Check solutions and reject any derived roots.

3. Solving Inequalities:

Absolute value	a. Same as solving absolute value equation; b. Use points to test area, write conjunction: $< x <$, or disjunction: $x <$ or $x >$.
Quadratic	a. Same as solving a quadratic equation; b. Use points to test area, write conjunction: $< x <$, or disjunction: $x <$ or $x >$.

4. On Calculator:

- a. Intersection Points:
 $Y =$ (Plug in equations) 2^{nd} TRACE 5 (Move cursor to the point of intersection) ENTER (3 times)
- b. Zeroes:
 2^{nd} TRACE 2 (Move cursor to left of x-intercept) ENTER (Move cursor to right of x-intercept) ENTER (twice)

QUADRATIC EQUATIONS

1. Finding Roots of a Quadratic Equation:

The answers to a quadratic equation are called roots. Graphically these roots are the #s on x-axis. To find these roots, set the equation equal to zero, then solve the equation using either factoring or quadratic

formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. Finding the Nature of Roots:

Use the discriminant $b^2 - 4ac$ to find:

If the discriminant is	The roots will be
A negative number	Imaginary (never crosses x-axis)
Zero	Equal (graph crosses x-axis once)
Positive perfect square	Rational but unequal (graph crosses x-axis twice)
Positive nonperfect square	Irrational and unequal (graph crosses x-axis twice)

3. Finding the Sum and Product of Roots:

When the equation is in standard form: $ax^2 + bx + c = 0$, use formulas: sum = $-b/a$, product = c/a .

4. Writing a Quadratic Equation:

- Find the sum of 2 roots;
- Find the product of the 2 root;
- substitute these into: $x^2 + (\text{inverse of sum})x + (\text{product}) = 0$.

5. Finding the Turning Point of a Parabola:

For $y = f(x) = ax^2 + bx + c$:

Axis of symmetry	$x = \frac{-b}{2a}$
Vertex (turning point)	$(\frac{-b}{2a}, f(\frac{-b}{2a}))$ (use $x = \frac{-b}{2a}$ to find x, then substitute into equation to find y)
Direction of opening	Upward if $a > 0$, downward if $a < 0$

6. Equation of a circle:

$$(x - h)^2 + (y - k)^2 = r^2 \quad [\text{center } (h, k), r = \text{radius}]$$

FUNCTIONS

1. Determining If it is a Function:

- In ordered pairs: no two pairs can have the same value of x;
- In a graph: no vertical line can intersect the graph more than once.

2. f(x) Notation:

To evaluate $f()$; substitute whatever is in the $()$ into x. This can only give one possible answer, since it is a function.

3. Domain and Range:

- Domain: the values of x used, usually written as: $\leq x \leq$ (From a graph, it is the width)
- Range: the values of y used, usually written as: $\leq y \leq$ (From a graph, it is the height)

4. Finding Domain Given As an Equation:

- Fractions: set the denominator $\neq 0$ and solve;
- Square roots: set what is underneath the $\sqrt{\quad}$, ≥ 0 and solve;
- Square root in denominator of fraction: set what is underneath the $\sqrt{\quad}$, > 0 and solve.

5. Inverse of a Function ($f^{-1}(x)$):

- Graphically, the inverse is a reflection over the line $y = x$;
- When given ordered pairs: switch x and y in each pair;
- When given an equation: switch x and y in the equation and rewrite the equation into $y =$.

6. One-To-One Function:

Both self and its inverse are functions. Every one-to-one function passes both vertical and horizontal line test.

7. $(f \circ g)(x)$ Notation:

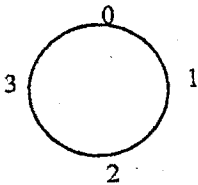
This is called the composition of functions. Start with the right one and move left. Whatever is in the $()$ gets substituting into x of the next function.

8. Inverse Variation:

$y = k/x$, as x increases, y decreases. Each time x and y multiply, the product is the same. When graphed, it looks like a rectangular hyperbola.

COMPLEX NUMBERS

1. Powers of i :



$$\begin{aligned}i^0 &= 1 \\i^1 &= i \\i^2 &= -1 \\i^3 &= -i\end{aligned}$$

Divide the exponent by 4 and match the remainder with the chart.

2. Simplifying Imaginary Numbers:

$\sqrt{-1} = i$. For any real number b , where $b > 0$: $\sqrt{-b^2} = \sqrt{b^2}\sqrt{-1} = bi$

3. Additive Inverse: $a + (-a) = 0$

Same 2 terms, both with the opposite sign. (Remember: conjugate only changes the one sign between terms).

4. Multiplicative Inverse: $a(1/a) = 1$

- Take the reciprocal;
- Multiply by conjugate to get rid of i in denominator.

5. Combining Complex Numbers:

Simplify any $\sqrt{\quad}$ and use i to replace " $\sqrt{\quad}$ " sign, combine terms without i , and terms with i .

6. Adding Complex Numbers Graphically:

The horizontal axis is the real axis, the vertical axis is the imaginary axis. Each complex number is graphed as a vector connected to the origin. Graph the 2 original complex numbers: connect the point (a, b) of a complex number $a + bi$ with the origin. Their sum is the diagonal of the parallelogram formed by them.

7. Absolute Value of Complex Number:

This represents its distance from the origin. Use formula below or Pythagorean Theorem:

$$|a + bi| = \sqrt{a^2 + b^2}$$

EXPONENTS AND LOGS

EXPONENTS

1. Negative Exponents: $x^{-n} = \frac{1}{x^n}$

2. Zero Exponent: $x^0 = 1$

3. Fractional Exponents:

$x^{a/b} = b\sqrt[b]{x^a}$ or $(b\sqrt[b]{x})^a$ Remember: $x^{1/2} = \sqrt{x}$, $x^{1/3} = \sqrt[3]{x}$

4. Exponent Equations:

a. If exponent is a number: raise both sides to the reciprocal of the given exponent;

(Ex. $x^{2/3} = 9 \rightarrow (x^{2/3})^{3/2} = 9^{3/2} \rightarrow x = 27$)

b. If exponent has a variable: factor the bases to the same base, solve the equation the exponents create.

If the bases cannot be rewritten as the same, use log(see "log").

(Ex. $8^x = 4^{x+3} \rightarrow 2^{3(x)} = 2^{2(x+3)} \rightarrow 3x = 2(x+3) \rightarrow x = 6$)

LOGS

1. Relationship of Log and Exponent:

$\log_a x = y \rightarrow x = a^y$ (Note: to get rid of "log", use the same base and interchange x and y.)

2. Common Log:

The base of log is 10: $\log x = y \rightarrow x = 10^y$ (a = 10)

3. Laws of Logs:

Product rule: $\log_b xy = \log_b x + \log_b y$

Quotient rule: $\log_b x/y = \log_b x - \log_b y$

Power rule: $\log_b x^c = c \log_b x$

4. Logs on Calculator:

a. Calculator accepts common log only:

LOG (#) ENTER

b. If the base is not 10:

Rewrite the log into common log: $\log_b a = \frac{\log a}{\log b}$, then use calculator to divide.

5. Using Logs to Solve Exponent Equations:

If exponent has a variable:

Ex. $12 \cdot 12^x = 500$
 $\log(12 \cdot 12^x) = \log 500$
 $\log 12 + x \log 12 = \log 500$
 $x = (\log 500 - \log 12) / \log 12$

a. "Log" both sides;

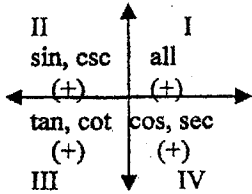
b. Apply log Laws;

c. Solve for x.

TRIGONOMETRY

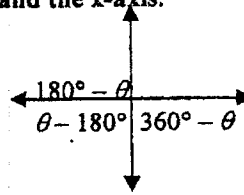
BASIC TRIG. INFORMATION

1. Sign of Functions in each Quadrant:
ASTC (All Students Take Calculus)



2. Reference Angles:

An acute angle formed by the terminal side of the given angle and the x-axis.



3. Function Values of Special Angles and Quadrantal Angles:

θ	0°	30°	45°	60°	90°	180°	270°	360°
$\sin \theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0	-1	0
$\cos \theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	-1	0	1
$\tan \theta$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	undefined	0	undefined	0

4. Radian \leftrightarrow Degree Measure:

Measure of an angle in radians = length of the intercepted arc/length of the radius: $\theta = s/r$

π radians = 180° degree \rightarrow radian: multiply by $\pi/180^\circ$; radian \rightarrow degree: replace π with 180° .

(On calculator, make sure to change MODE into RADIAN or DEGREE.)

5. Coterminal Angles:

Two angles in standard position with same terminal side. (e.g. $300^\circ \leftrightarrow -60^\circ$.)

6. Cofunctions:

Two trig. functions such that the trig. function of an acute angle is equal to the cofunction of its complement, $90^\circ - \theta$.

$$\sin A = \cos B$$

$$\tan A = \cot B$$

$$\sec A = \csc B$$

$$\longrightarrow A + B = 90^\circ$$

7. Eight Basic Trigonometric Identities:

Reciprocal Identities	Quotient Identities	Pythagorean Identities
$\csc \theta = \frac{1}{\sin \theta}$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$	$\sin^2 \theta + \cos^2 \theta = 1$
$\sec \theta = \frac{1}{\cos \theta}$	$\cot \theta = \frac{\cos \theta}{\sin \theta}$	$1 + \tan^2 \theta = \sec^2 \theta$
$\cot \theta = \frac{1}{\tan \theta}$		$1 + \cot^2 \theta = \csc^2 \theta$

TRIG. GRAPHS

1. Graphs of Sinx and Cosx:

a.

b.

2. Amplitude, Frequency, and Period of Graphs $y = a\sin bx$ or $y = a\cos bx$:

- Amplitude: $|a|$ (Maximum or minimum height of the graph above and under the x-axis.)
Frequency: $|b|$ (Number of complete curves that appear between 0 and 2π .)
Period: $2\pi/|b|$ (The place where the first complete curve finishes.)
Range: $-a \leq y \leq a$

3. For Graphs of $y = a\sin bx + c$ or $y = a\cos bx + c$:

Each graph has been moved up or down, changing with the middle line c .
[Also found by calculating: $c = (\text{max. height} + \text{min. height})/2$]

4. Writing an Equation for a Given Graph:

- Determine if it is sin or cos (sin starts at midline, cos starts at amplitude);
- Find midline c $\{(\text{max.} + \text{min.})/2\}$;
- Find amplitude a $\{(\text{max.} - \text{min.})/2\}$;
- Find frequency b (x-value where first complete curve ends), and period $2\pi/b$.
Always check the graph on calculator.

5. On Calculator:

MODE Radian/Degree ENTER

Y= (Plug in equation) ZOOM 7 (Each mark on the x-axis represents 90° or $\pi/2$ radians.)

TRIG. APPLICATIONS

1. Finding Area of a Triangle:

Area of $\triangle ABC$ $K = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$
(Remember: the angle used must be between the 2 sides.)

2. Law of Sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

3. Law of Cosines:

$$a^2 = b^2 + c^2 - 2bccosA$$

4. Finding 1 Missing Part of a Triangle:

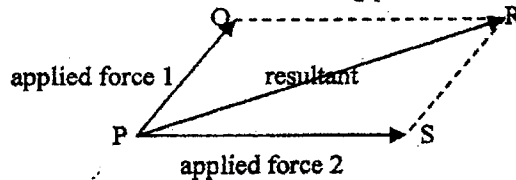
- Draw \triangle and fill in given information, including the missing part;
- See how many opposite side/angle pairs,
if: 2 pairs --- use Law of Sines;
1 pair --- use Law of Cosines.

5. Ambiguous Case:

Always use Law of Sines to find $\angle B$ first. Use $\angle A$ and $\angle B$ to find how many $\angle C$. The number of possible triangles is the number of angle $\angle C$.

6. Forces:

Construct a parallelogram using 2 given applied forces as 2 sides, the diagonal is resultant force. Use Law of Sines or Law of Cosines to find missing part.



Note: Never use the angle between 2 forces.

TRIG. EQUATIONS

1. Make sure the equation contains only 1 trig. function. If not, substitute using trig. formulas;
2. If equation is quadratic, use factoring, or quadratic formula;
3. Find all solutions that are within the interval provided by considering reference angles and quadrants.

PROBABILITY

1. Permutations vs. Combination: ${}_n P_r$ or ${}_n C_r$

Permutation: an arrangement of objects in some specific order.
 Combination: a collection of objects without order.

2. Permutation with Repetition in a Word:

To find the total possible number of arrangements of letters in a word:

$$\frac{n!}{a! b! c!} \quad (n = \text{total \# of letters in a word, } a! b! c! = \text{\# of times each letter repeats.})$$

3. Bernoulli's Theorem: (binomial probability)

$$P = {}_n C_r p^r q^{n-r}$$

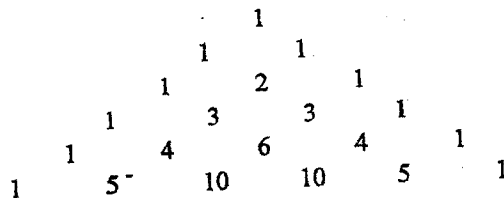
$(n = \text{total number of trials, } r = \text{number of successes, } n - r = \text{number of failures, } p = \text{probability of success, } q = 1 - p = \text{probability of failure, } p + q = 1)$

In calculator: 2nd DISTR binompdf ENTER (n, p, r)

At least r successes in n trials means: $r, r + 1, r + 2, \dots, n$ successes.
 At most r successes in n trials means: $r, r - 1, r - 2, \dots, 0$ successes.

4. Pascal's Triangle.

$$\begin{aligned} (x+y)^0 &= 1 \\ (x+y)^1 &= 1x + 1y \\ (x+y)^2 &= 1x^2 + 2xy + 1y^2 \\ (x+y)^3 &= 1x^3 + 3x^2y + 3xy^2 + 1y^3 \\ (x+y)^4 &= 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4 \\ (x+y)^5 &= (\text{to be answered}) \end{aligned}$$



$$(x+y)^n = {}_n C_0 x^n y^0 + {}_n C_1 x^{n-1} y^1 + {}_n C_2 x^{n-2} y^2 + \dots + {}_n C_{n-1} x^1 y^{n-1} + {}_n C_n x^0 y^n$$

5. Specific Term of a Binomial Expansion:

The r^{th} term of the expansion is: ${}_n C_{r-1} x^{n-r+1} y^{r-1}$.

Remember:

1. For any binomial expansion $(x+y)^n$, there are $n + 1$ terms after expanded;
2. x and y include positive and negative numbers.

STATISTICS

1. Summation " Σ ":

e.g. $\sum_{k=0}^5 3^k = 3^0 + 3^1 + 3^2 + 3^3 + 3^4 + 3^5 = 364$

2. Central Tendency:

Use calculator to find $\sum x$, mean (\bar{x}), median (Med), range ($\max X - \min X$), standard deviation (σ_x):

- Clear all lists: 2nd MEM 4 ENTER or above L1(↑): CLEAR ENTER
- Enter data: STAT EDIT enter data under L1 (If frequencies, enter under L2)
- Find $\sum x$, \bar{x} , Med, $\max X$, $\min X$, σ_x : STAT CALC ENTER ENTER
- Find σ_x of L1 and L2: STAT CALC ENTER 2nd L1 , 2nd L2 ENTER

3. Normalized or Standardized Data:

Refer to "Normal Distribution" curve on the formula sheet.

Remember: the horizontal scale is for every $\frac{1}{2}$ of a Standard Deviation. Percentiles are found by adding up all the %s at or below the score given.

4. Correlation Coefficient (r):

- STAT ENTER: Enter data in L₁ and L₂
- 2nd STAT PLOT ENTER ENTER TYPE(scatter plot) ENTER ZOOM 9
- 2nd CATALOG DiagnosticOn ENTER ENTER
- STAT CALC 4 2nd L1 , 2nd L2 ENTER: r

5. Regression Line:

a. Drawing regression line:

STAT EDIT: Enter data in L₁ and L₂

Y= VARS 5 EQ ENTER ZOOM 9

b. Writing Regression line equation:

STAT EDIT: Enter data in L₁ and L₂

STAT CALC LinReg/ExpReg/PowReg (based on what you are looking for) ENTER

c. Using equation to predict:

STAT EDIT: Enter data in L₁ and L₂

Y= VARS 5 EQ ENTER ZOOM 9

2ND CALC ENTER (value) ENTER

(Change WINDOW to be greater than the value entered so that the required value will be shown.)

Sequence and Series

A **sequence** is a set of numbers written in a given order. Each term of a sequence is associated with the positive integer that specifies its position in the ordered set. A **finite sequence** is a function whose domain is the set of integers $\{1, 2, 3, \dots, n\}$. An **infinite sequence** is a function whose domain is the set of positive integers. The terms of a sequence are often designated as a_1, a_2, a_3, \dots . The formula that allows any term of a sequence except the first to be computed from the term is called **recursive definition**.

e.g.

Write the first five terms of the sequence if $a_1 = 5, a_n = a_{n-1} + 1$

$$a_2 = a_{2-1} + 1 = a_1 + 1 = 5 + 1 = 6$$

$$a_3 = a_{3-1} + 1 = a_2 + 1 = 6 + 1 = 7$$

$$a_2 = a_{2-1} + 1 = a_1 + 1 = 5 + 1 = 6$$

An **arithmetic sequence** is a sequence such that for all n , there is a constant d such that $d = a_{n+1} - a_n$. For an arithmetic sequence: $a_n = a_1 + d(n-1)$

A **geometric sequence** is a sequence such that for all n , there is a constant r such that $r = \frac{a_{n+1}}{a_n}$. For a geometric

sequence: $a_n = a_1 r^{n-1}$

A **series** is the indicated sum of the terms of a sequence. The Greek letter Σ is used to indicate a sum of defined for a set of consecutive integer.

If S_n represents the n^{th} **partial sum**, the sum of the first n terms of a sequence, then

$$S_n = \sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots + a_n$$

For an arithmetic series: $S_n = \frac{n}{2}(a_1 + a_n)$

For a finite geometric series: $S_n = \frac{a_1(1-r^n)}{1-r}$

For an infinite geometric series, if $-1 < r < 1$ and n approaches infinity: $S_n = \frac{a_1}{1-r}$