Aim: How do we rationalize a denominator containing a radical?

Do Now: 1. Find all the irrational numbers:

$$\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{7}, \sqrt{8}, \sqrt{16}, \sqrt{30}, \sqrt{20}$$

2. Simplify: $\dfrac{1}{\sqrt{4}}$

3. Simplify: $\dfrac{1}{\sqrt{5}}$

We can only simplify $\dfrac{1}{\sqrt{4}}$ since $\sqrt{4}$ is rational number.

What happen to $\dfrac{1}{\sqrt{5}}$?

We cannot simplify $\dfrac{1}{\sqrt{5}}$ by using regular method

since the denominator $\sqrt{5}$ is an irrational number.

Therefore, we need to do something to make the denominator become a rational number.

This procedure is called rationalize the denominator (where the denominator is not a rational number) means to find an equivalent fraction in which the denominator is a rational number.

How do we rationalize the irrational denominator?

1. Multiply the irrational denominator by the exact same irrational number to get square and then get rid of the square root

$$\frac{1 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}}$$

2. Multiply the numerator by the number as the denominator did

$$\frac{\sqrt{5}}{\sqrt{5}^{2}}$$

3. Simplify the resulting fraction

$$\frac{\sqrt{5}}{5}$$

Rationalize: $\dfrac{9}{3-\sqrt{3}}$

1. Multiply the denominator by its conjugate which is $3+\sqrt{3}$

$$\frac{9}{(3-\sqrt{3})(3+\sqrt{3})}$$

2. Multiply the numerator by the same number as the denominator did

$$\frac{9(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})}$$

3. Simplify the resulting fraction

$$\frac{27+9\sqrt{3}}{9-\sqrt{3}^{2}} = \frac{27+9\sqrt{3}}{6} = \frac{9+3\sqrt{3}}{2}$$

To rationalize the radical denominator, we need to multiply the denominator by its conjugate

The reason why we multiply the denominator by its conjugate is to get the difference of two squares, therefore we can get rid of the radical.

Conjugate

x - y and x + y are conjugates

$$(x - y)(x + y) = x^2 - y^2$$

Rationalize: $\dfrac{9}{\sqrt{11}} = \dfrac{9\sqrt{11}}{\sqrt{11}\cdot\sqrt{11}} = \dfrac{9\sqrt{11}}{\sqrt{11}^{2}} = \dfrac{9\sqrt{11}}{11}$

Rationalize: $\dfrac{3-\sqrt{2}}{5+\sqrt{2}} = \dfrac{(3-\sqrt{2})(5-\sqrt{2})}{(5+\sqrt{2})(5-\sqrt{2})}$

$$= \frac{15-3\sqrt{2}-5\sqrt{2}+\sqrt{2}^{2}}{5^{2}-\sqrt{2}^{2}}$$

$$= \frac{15-8\sqrt{2}+2}{25-2} = \frac{17-8\sqrt{2}}{23}$$

Find the sum $\frac{\sqrt{2}}{3-\sqrt{6}}+\frac{6}{\sqrt{8}}$

$$=\frac{\sqrt{2}(3+\sqrt{6})}{(3-\sqrt{6})(3+\sqrt{6})}+\frac{6\sqrt{8}}{\sqrt{8}^{2}}=\frac{3\sqrt{2}+\sqrt{12}}{9-6}+\frac{6\cdot 2\sqrt{2}}{8}$$

$$=\frac{3\sqrt{2}+2\sqrt{3}}{3}+\frac{12\sqrt{2}}{8}=\frac{3\sqrt{2}+2\sqrt{3}}{3}+\frac{3\sqrt{2}}{2}$$

$$=\frac{6\sqrt{2}+4\sqrt{3}}{6}+\frac{9\sqrt{2}}{6}=\frac{15\sqrt{2}+4\sqrt{3}}{6}$$

Rationalize the denominator

1. $\dfrac{40}{2\sqrt{20}}$ $2\sqrt{5}$

2. $\dfrac{18}{\sqrt{3} - 3}$ $-3(\sqrt{3} + 3)$

3. $\dfrac{\sqrt{2} + 4}{\sqrt{2} - 1}$ $6 + 5\sqrt{2}$