Aim: How do we solve radical equations?

- **Do Now**: Solve for *x*:
- 1. $x + 1 = 5$
- 2. $x + 1 = -5$
- 3. $\sqrt{x+1} - 5 = 0$
- 4. $\sqrt{x+1} + 5 = 0$

HW: p.112 # 16,18,22,24,26,28

p. 37 # 29,31 p.35 # 8 p.115 # 20

Radical Equations

An equation in which a variable occurs in the radicand is called a radical equation. It should be noted, that when solving a radical equation algebraically, extraneous roots may be introduced when both sides of an equation are squared. Therefore, you must check your solutions for a radical equation.

Solve: $\sqrt{x+1} - 5 = 0$ $\quad x \geq 3$

$$\sqrt{x+1} = 5$$

$$(\sqrt{x+1})^2 = (5)^2$$

$$x + 1 = 25$$

$$x = 24$$

Check:

L.S.	R.S.
$\sqrt{x+1} - 5$	0
$\sqrt{24+1} - 5$	
$5 - 5$	
0	

Therefore, the solution is $x = 24$.

$$\sqrt{x+1}+5=0$$

$$\sqrt{x+1}=-5$$

$$(\sqrt{x+1})^2=(-5)^2$$

$$x+1=25$$

$$x=24$$

Check: $\sqrt{24+1}+5=0$

$$\sqrt{25}+5=0$$

$$5+5=0$$

$$10 \neq 0$$

reject the answer

$x = 24$ is **extraneous**.

Therefore, there are no real roots.

Solving Radical Equations

$$4 + \sqrt{4 + x^2} = x$$

$$\sqrt{4 + x^2} = x - 4$$

$$(\sqrt{4 + x^2})^2 = (x - 4)^2$$

$$4 + x^2 = x^2 - 8x + 16$$

$$8x = 12$$

$$x = \frac{3}{2}$$

Check:

$4 + \sqrt{4 + x^2}$	x
$4 + \sqrt{4 + \left(\frac{3}{2}\right)^2}$	$\frac{3}{2}$
$4 + \sqrt{4 + \frac{9}{4}}$	
$4 + \sqrt{\frac{25}{4}}$	
$4 + \frac{5}{2}$	
$\frac{13}{2}$	$\neq \frac{3}{2}$

Since $\frac{3}{2} \neq \frac{13}{2}$, the solution of $x = \frac{3}{2}$ is extraneous. Therefore, there are no real roots.

Solving Radical Equations

Solve $\sqrt{2x+4} - \sqrt{x+7} = 0.$ $x \geq -2$

$$\sqrt{2x+4} = \sqrt{x+7}$$

$$\left(\sqrt{2x+4}\right)^2 = \left(\sqrt{x+7}\right)^2$$

$$2x + 4 = x + 7$$

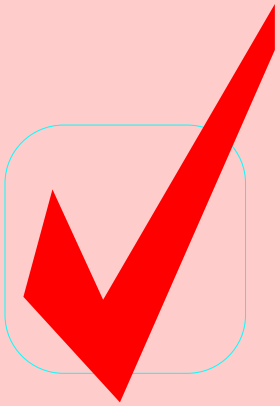
$$x = 3$$

Set up the equation so that there will be one radical on each side of the equal sign.

Square both sides.

Simplify.

L.S.	R.S.
$\sqrt{2x+4} - \sqrt{x+7}$	0
$\sqrt{2(3)+4} - \sqrt{3+7}$	
$\sqrt{10} - \sqrt{10}$	
0	



Verify your solution.

Therefore, the solution is $x = 3$.

Solving Radical Equations

Solve $\sqrt{5x+1} - \sqrt{3x-5} = 2.$

Set up the equation so that there will be only one radical on each side of the equal sign.

$$\sqrt{5x+1} = 2 + \sqrt{3x-5}$$

Square both sides of the equation.

$$\left(\sqrt{5x+1}\right)^2 = \left(2 + \sqrt{3x-5}\right)^2$$

Use Foil.

$$5x + 1 = 4 + 4\sqrt{3x-5} + (3x-5)$$

Simplify.

$$5x + 1 = 3x - 1 + 4\sqrt{3x-5}$$

Simplify by dividing by a common factor of 2.

$$2x + 2 = 4\sqrt{3x-5}$$

$$x + 1 = 2\sqrt{3x-5}$$

Square both sides of the equation.

$$(x+1)^2 = \left(2\sqrt{3x-5}\right)^2$$

Use Foil.

$$x^2 + 2x + 1 = 4(3x-5)$$

Solving Radical Equations

$$x^2 + 2x + 1 = 4(3x - 5)$$

$$x^2 + 2x + 1 = 12x - 20$$

$$x^2 - 10x + 21 = 0$$

$$(x - 3)(x - 7) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x - 7 = 0$$

$$x = 3 \quad \text{or} \quad x = 7$$

Distribute the 4.

Simplify.

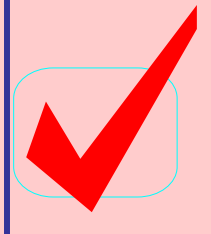
Factor the quadratic.

Solve for *x*.

Verify both solutions.

L.S.	R.S.
$\sqrt{5x+1} - \sqrt{3x-5}$	2
$\sqrt{5(3)+1} - \sqrt{3(3)-5}$	
$4 - 2$	
2	

L.S.	R.S.
$\sqrt{5x+1} - \sqrt{3x-5}$	2
$\sqrt{5(7)+1} - \sqrt{3(7)-5}$	
$6 - 4$	
2	



One more to see another extraneous solution:

$$\sqrt{3x+1} = x - 3$$

The radical is already isolated

$$\left(\sqrt{3x+1}\right)^2 = (x-3)^2$$

Square both sides

You must square the whole side NOT each term.

This must be FOILed

$$3x + 1 = x^2 - 6x + 9$$

$$x^2 - 9x + 8 = 0$$

$$(x-8)(x-1) = 0$$

$x = 8$, ~~$x = 1$~~

checks!

$$\sqrt{3(8)+1} = 8 - 3$$

$$5 = 5$$

$$\sqrt{3(1)+1} = 1 - 3$$

$$2 = -2$$

Therefore, the solution is 8 only.

Drill

1. $\sqrt{5x-1}-3=0$ **x = 2**

2. $\sqrt{x^2+9}-x=1$ **x = 4**

3. $\sqrt{5x+6}=\sqrt{9x-2}$ **x = 2**

4. $\sqrt{3x-1}=2\sqrt{8-2x}$ **x = 3**

5. $\sqrt{x^2-7}=x-1$ **x = 2**