Aim: What is the binomial theorem?

HW: p.711 # 3,12-17,19

Do Now: 1) Express each in simplest form:

a) $(x + y)^2$ b) $(x + y)^3$ c) $(x + y)^4$

Ans:

1)a) $(x + y)^2$

$(x + y)(x + y) = x^2 + 2xy + y^2$

b) $(x + y)^3$

$(x + y)(x + y)(x + y) = (x + y)(x^2 + 2xy + y^2) =$

$x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3 =$

$x^3 + 3x^2y + 3xy^2 + y^3$

c) $(x + y)^4$

$(x + y)(x + y)(x + y)(x + y) = (x + y)(x^3 + 3x^2y + 3xy^2 + y^3) =$

$= x^4 + 3x^3y + 3x^2y^2 + xy^3 + x^3y + 3x^2y^2 + 3xy^3 + y^4$

$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

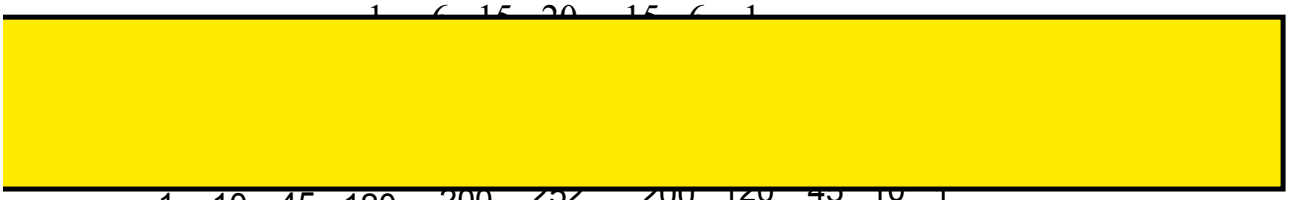
Recall of course that $(x + y)^0 = 1$ and $(x + y)^1 = x + y$

Let's take a look at the bigger picture:

$$1$$
$$1x + 1y$$
$$1x^2 + 2xy + 1y^2$$
$$1x^3 + 3x^2y + 3xy^2 + 1y^3$$
$$1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$$

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1



1 10 45 120 200 252 200 120 45 10 1

Show $(x + y)^{10}$

10 9 8 7 6 5 4 3 2 1 0

$x^{10}y^0+10x^9y^1+45x^8y^2+120x^7y^3+200x^6y^4+252x^5y^5+200x^4y^6+120x^3y^7+45x^2y^8+10x^1y^9+1x^0y^{10}$

Formula on Reference Sheet

$$(a + b)^n = {}_nC_0\, a^n b^0 + {}_nC_1\, a^{n-1} b^1 + {}_cC_2\, a^{n-2} b^2 \ldots {}_nC_n a^{n-n} b^n$$

$_nC_r$ n is the power of the binomial $(x + y)^n$
r is the specific term

For example: $(x + y)^2 = x^2 + 2xy + y^2$

the 1st coefficient $_2C_0 = 1$

the 2nd coefficient $_2C_1 = 2$

the 3rd coefficient $_2C_2 = 1$

Note: $_nC_0$ for the 1st term coefficient not $_nC_1$

In the expansion $(x + y)^n$, the first term is $x^n y^0$ or x^n

Then, in each successive term, the exponent of x decreases by 1 as the exponent of y increases by 1 until the last term, $x^0 y^n$ or just y^n, is reached.

Rule: For any binomial (x + y)n, there are n + 1 terms after expansion.

Example: Expand (x - y)5

Expand $(x - y)^5$

nCr x y

$$\overset{10}{x^{10}y^{0}}+\overset{9}{10x^{9}y^{1}}+\overset{8}{45x^{8}y^{2}}+\overset{7}{120x^{7}y^{3}}+\overset{6}{200x^{6}y^{4}}+\overset{5}{252x^{5}y^{5}}+\overset{4}{200x^{4}y^{6}}+\overset{3}{120x^{3}y^{7}}+\overset{2}{45x^{2}y^{8}}+\overset{1}{10x^{1}y^{9}}+\overset{0}{1x^{0}y^{10}}$$

Find the 5th term of the expansion $(2x - y)^8$

1st Find the nCr 0 -1-2-3-4 it is ${}_8C_4$
2nd count backwards to find the first term .... $(2x)^5(-y)^4$
3rd Evaluate ${}_8C_4\ (2x)^5(-y)^4 = (70)(32x^5)y^4$

You may also find the specific term in an expansion by a formula.

$$ {}_nC_{r-1}\, x^{n-r+1} y^{r-1} $$

Find the 5th term of the expansion $(2x - y)^8$

Example: Find the middle term of $(x-2)^6$

The middle term is the 4^{th} term

The coefficient is ${}_6C_3 = 20$

The exponent is $x^3(-2)^3 = -8x^3$

$$20 \cdot -8x^3 = -160x^3$$

Write the expansions of:

1) $(b^2 - 3)^3$

2) $(2k - 1)^4$

3) Challenge $(2a^2 - b^3)^3$

Answers:

1) $b^6 - 9b^4 + 27b^2 - 27$

2) $16k^4 - 32k^3 + 24k^2 - 8k + 1$

3) $8a^6 - 12a^4b^3 + 6a^2b^6 - b^9$

$$\begin{array}{ccccccccccc} 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{array}$$
$$x^{10}y^{0}+10x^{9}y^{1}+45x^{8}y^{2}+120x^{7}y^{3}+200x^{6}y^{4}+252x^{5}y^{5}+200x^{4}y^{6}+120x^{3}y^{7}+45x^{2}y^{8}+10x^{1}y^{9}+1$$