

Lesson 71 MR12H

Aim: How do we prove and apply the Pythagorean identities?

☺ HW: Complete Worksheet ☺

Do Now: Find the value of

1. $\sin^2 30^\circ + \cos^2 30^\circ$

2. $\sin^2 45^\circ + \cos^2 45^\circ$

3. $\sin^2 60^\circ + \cos^2 60^\circ$

4. $\sin^2 \theta + \cos^2 \theta$

Test 9
Tuesday
3/15/11

Reciprocal Functions

HW: page 414 # 6,11,15 to 20

GB page pg 2 # 8,12,14

6. $\sin \theta = -\frac{2}{3}, \cos \theta = \frac{\sqrt{5}}{3}, \tan \theta = -\frac{2\sqrt{5}}{5},$
 $\cot \theta = -\frac{\sqrt{5}}{2}, \sec \theta = \frac{3\sqrt{5}}{5}, \csc \theta = -\frac{3}{2}$

11. $\sin \theta = -\frac{3\sqrt{34}}{34}, \cos \theta = -\frac{5\sqrt{34}}{34}, \tan \theta = \frac{3}{5},$
 $\cot \theta = \frac{5}{3}, \sec \theta = -\frac{\sqrt{34}}{5}, \csc \theta = -\frac{\sqrt{34}}{3}$

15. 1 16. $\cos \theta$ 17. $\sin \theta$

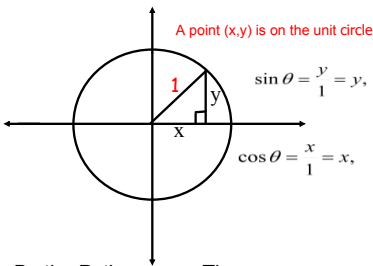
18. $\frac{1}{\sin \theta}$ 19. $\frac{1 + \sin \theta}{\cos \theta}$ 20. $\frac{\sin \theta}{\cos \theta}$

GB

8. (4)

12. (3)

14. (1)



By the Pythagorean Theorem
 $x^2 + y^2 = 1^2 = 1$

Replace x and y by $\cos \theta$ and $\sin \theta$, a new equation can be formed $(\cos \theta)^2 + (\sin \theta)^2 = 1$

or $\sin^2 \theta + \cos^2 \theta = 1$

This is the Pythagorean Identity.

An **identity** is an equation that is true for all values of the variable for which the terms of the variable are defined.

• How can the Pythagorean Identity be written in terms of the other 4 Trigonometric Functions?

• How can we change the equation to show a tangent function?

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

Pythagorean Identities

- $\sin^2 \theta + \cos^2 \theta = 1$
- $\cot^2 \theta + 1 = \csc^2 \theta$
- $\tan^2 \theta + 1 = \sec^2 \theta$

An **identity** is an equation that is true for all values of the variable for which the terms of the variable are defined.

What other Identities have we discovered?

Day 2 Aim: How do we use the Pythagorean Identities?

HW: Page 431

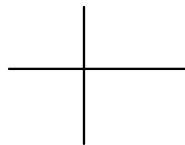
1,3,4,5,6,8,10,13,17,24,25,26,27

page 414 #10,21

Do Now:

csc B is 5 and this angle is in the 2nd Quadrant

Find the remaining 5 functions.



$$\sin B = \frac{\text{opp}}{\text{hyp}} = \quad \csc B = \frac{\text{hyp}}{\text{opp}} =$$

$$\cos B = \frac{\text{adj}}{\text{hyp}} = \quad \sec B = \frac{\text{hyp}}{\text{adj}} =$$

$$\tan B = \frac{\text{opp}}{\text{adj}} = \quad \cot B = \frac{\text{adj}}{\text{opp}} =$$

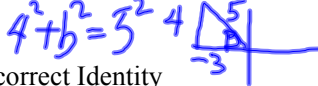
Test
Tuesday

$$\textcircled{1} \frac{\sin^2 x + \cos^2 x}{\cos x} = \frac{1}{\cos} = \sec \checkmark$$

$$\textcircled{2} \frac{4}{\sec^2 \theta} = \cos^2$$

How can we solve the same problem using the Pythagorean Identities?

csc B is $\frac{5}{4}$ and this angle is in the 2nd Quadrant



- 1st Select the correct Identity
- 2nd Substitute the function into the identity
- 3rd Find the missing functions.

→ $1 + \cot^2 B = \csc^2 B$

$$\begin{aligned}
 1 + \cot^2 B &= \left(\frac{5}{4}\right)^2 - 1 \\
 -1 & \quad -1 \\
 \cot^2 B &= \left(\frac{5}{4}\right)^2 - 1 \\
 \cot^2 B &= \frac{25}{16} - 1 \\
 &= \frac{25}{16} - \frac{16}{16} \\
 \sqrt{\cot^2 B} &= \sqrt{\frac{9}{16}} \\
 \cot B &= \frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 \sin B &= \frac{4}{5} \\
 \csc B &= \frac{5}{4} \\
 \cos B &= -\frac{3}{5} \\
 \sec B &= -\frac{5}{3} \\
 \tan B &= -\frac{4}{3} \\
 \cot B &= -\frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 \csc &= \frac{5}{4} \\
 \sin &= \frac{4}{5} \\
 \cos &= -\frac{3}{5} \\
 \sec &= -\frac{5}{3}
 \end{aligned}$$

- How do we use the Pythagorean Identities to solve problems?

Given $\sin A = .6$ in quadrant II, Find $\cos A$ using a Pythagorean Identity.

$$\begin{aligned}
 \sin A &= .6 = \frac{6}{10} \\
 \sin^2 A + \cos^2 A &= 1 \\
 \left(\frac{6}{10}\right)^2 + \cos^2 A &= 1 \\
 \frac{36}{100} + \cos^2 A &= 1 - \frac{36}{100} \\
 -\frac{36}{100} & \quad -\frac{36}{100} \\
 \cos^2 A &= \frac{100}{100} - \frac{36}{100} = \frac{64}{100} \\
 \sqrt{\cos^2 A} &= \sqrt{\frac{64}{100}} \\
 \cos A &= \frac{8}{10} = \frac{4}{5}
 \end{aligned}$$

$$\begin{aligned}
 &4. (\cot \csc^2 x) - (\cot x) \\
 &\cot x (\csc^2 x - 1) \\
 &\cot x (\cot^2 x) \\
 &\cot^3 x
 \end{aligned}$$

$$\begin{aligned}
 \cos(1 + \tan^2 \theta) &= \cos(\sec^2) \\
 \cos\left(\frac{1}{\cos^2}\right) &= \frac{1}{\cos} \\
 &= \sec
 \end{aligned}$$

$$\sin \theta = a$$

$$\cos^2 = 1 - \sin^2$$

$$\sqrt{\cos^2} = \sqrt{1 - a^2}$$

$$\cos = \sqrt{1 - a^2}$$

using a Pythagorean Identity.

1. $\cos A = \frac{\sqrt{3}}{2}$ A in quad. IV, find $\sin A$
2. $\tan A = \frac{-3}{4}$ A in IV, find $\sec A$
3. $\sec \theta = -2$, θ in III, find $\tan \theta$
4. $\cot \theta = 1$, θ in III, find $\csc \theta$

Simplify the following using the Pythagorean Identities.

5. $\sec A \cdot \cot A$

6. $\cot^2 A \cdot \tan A$

7. $\sec^2 A + \csc^2 A$

8. $\frac{\tan A}{\sec A}$

9. $\frac{1}{\sec A} (\tan A + \sec A)$

10. $\frac{\sec A - \csc A}{\sec A}$