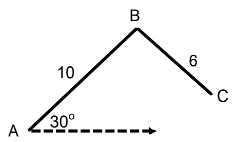
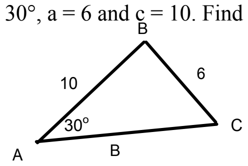
Aim: How do we handle the ambiguous case (SSA)?

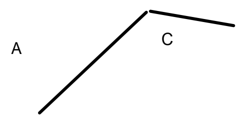
Do Now: Given AB = 10 inches , BC = 6 inches
Angle A = 30°
How many different triangles can you draw?
Remember we don't know the length of AC.



1) In ΔABC m ∠A = 30°, a = 6 and c = 10. Find m∠C to the nearest degree.



Angle C is 56. Since this is an SSA situation could Angle C be another angle?



180 - 56 = 124

Check 124 + 30 < 180
154 < 180
180 - 154 = 26
Another triangle with the angles of {26,30, 124}
besides {30,56, 94}

THUS the AMBIGUOUS CASE!!!!

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2) If in ΔABC a = 15, m ∠ A = 30° and c =12
a) find m∠C
b) How many triangles may be drawn?

Answer:

$$\frac{15}{\sin 30^\circ} = \frac{12}{\sin C}$$

$$\frac{15}{\frac{1}{2}} = \frac{12}{\sin C}$$

$$15 \sin C = \tfrac{1}{2}(12)$$
$$15 \sin C = 6$$
$$\sin C = 6/15$$
$$\sin C = .4$$
so m∠C = 24° or 156°

So that sort of answers part a) however we must remember that the given angle was 30° and 156° + 30° = 186° and that is too much for one triangle to hold. This means that for a) only 24° is a valid answer and for b) we can draw only ONE triangle.

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3) If in ΔABC, $a = 4$, $m\angle A = 30$, and $c = 12$
a) find $m\angle C$
b) how many triangles can be drawn?

Answer:

$$\frac{4}{\sin 30°} = \frac{12}{\sin C}$$

$$\frac{4}{\frac{1}{2}} = \frac{12}{\sin C}$$

$$4 \sin C = \tfrac{1}{2}(12)$$
$$4 \sin C = 6$$
$$\sin C = 6/4$$
$$\sin C = 1.5$$

a) Since $\sin C = 1.5$ we cannot have an angle measure for C so for a)
$m\angle C = \{ \}$

b) Since there is NO measure for $\angle C$ we cannot draw any triangles so for b) we say there are 0 triangles that can be drawn.

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4) If in ΔABC $a= 8$, $m\angle A = 150°$, and $c = 12$
a) find $m \angle C$
b) how many triangles can be drawn?

ANSWER:

$$\frac{8}{\sin 150°} = \frac{12}{\sin C}$$

$$\frac{8}{\frac{1}{2}} = \frac{12}{\sin C}$$

$$8 \sin C = \tfrac{1}{2}(12)$$
$$8 \sin C = 6$$
$$\sin C = 6/8$$
$$\sin C = .75$$
$$m\angle C = 49° \text{ or } 131°$$

a) If the given angle is 150° then $150° + 49° = 199°$ too much for 1 and then certainly $150° + 131°$ is way too much so neither answer is good and then the answer for b) is that there are 0 triangles that can be drawn.

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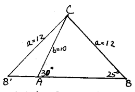
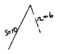


APPLICATIONS:

1. How many different triangles can be constructed if $m\angle R = 45°$, $r = 6$, $s = 10$?
2. In triangle LMN, $m\angle L = 55°$, $n = 8$, $\ell = 7$. Find $m\angle N$ to the nearest degree.
3. In Δ ABC, $a = 10$, $b = 11$, $\sin \angle A = 0.8660$. Find $m\angle B$ to the nearest ten minutes.
4. If, in Δ ABC, $m\angle A = 30$, $a = \sqrt{5}$ and $b = 4$, then $\angle B$
   a) may be either obtuse or acute
   b) must be obtuse, only
   c) must be acute, only
   d) may be a right angle

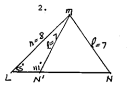
SOLUTIONS TO APPLICATION PROBLEMS:

1.



$$\frac{\sin \angle S}{s} = \frac{\sin \angle R}{r} =$$
$$\sin \angle S = \frac{s \sin \angle R}{r} = \frac{10 \sin 45°}{6}$$
$$= 1.1785$$

2.



$$\frac{\sin \angle N}{n} = \frac{\sin \angle L}{\ell} =$$
$$\sin \angle N = \frac{n \sin \angle L}{\ell} = \frac{8 \sin 55°}{7} = 0.9362$$

Also $n > \ell$, therefore $\angle N > \angle L$.

$m\angle N = 69°$ or $111°$

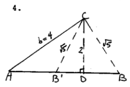
TWO SOLUTIONS ARE POSSIBLE.

3. $\frac{\sin \angle B}{11} = \frac{0.8660}{10}$

   $\sin \angle B = 0.9526$

   $m \angle B \approx 72°20'$ or $180° - 72°20' = 107°40'$

4.



First find CD=2 (30°-60° rt. Δ)
since $2 < \sqrt{5} < 4$, two triangles are possible and choice (a) is correct.

or

$$\frac{\sin A}{a} = \frac{\sin B}{b};$$
$$\sin B = \frac{4 \sin 30}{\sqrt{5}} = \frac{2}{\sqrt{5}} = .8944$$

$a < b$, so $\angle A < \angle B$
and $\angle B$ may be $63°$ or $117°$

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