Pre-Calculus - Honors — Lesson 26

Worksheet #26 — ***Answers***

1) $g(x) = x^3 - 4x^2 - x + 4$ — 2 variations = 2 or 0 positive real roots

$g(-x) = -x^3 - 4x^2 + x + 4$ — 1 variation = 1 negative real root

$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 4}{\pm 1}$ — The x-intercepts appear at -1, 1, 4.

test:

-1	1	-4	-1	4
		-1	5	-4
	1	-5	4	0

1	1	-4	-1	4
		1	-3	-4
	1	-3	-4	0

4	1	-4	-1	4
		4	0	-4
	1	0	-1	0

$x = \{-1, 1, 4\}$

2) $h(x) = x^3 - 9x^2 + 20x - 12$ — 3 variations = 3 or 1 positive real roots

$h(-x) = -x^3 - 9x^2 - 20x - 12$ — 0 variations = 0 negative real roots

$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1}$ — The graph shows x-intercepts at 1, 2, 6.

test:

1	1	-9	20	-12
		1	-8	12
	1	-8	12	0

2	1	-9	20	-12
		2	-14	12
	1	-7	6	0

6	1	-9	20	-12
		6	-18	12
	1	-3	2	0

$x = \{1, 2, 6\}$

3) $h(x) = x^3 + 12x^2 + 21x + 10$ — 0 variations = 0 positive real roots

$h(-x) = -x^3 + 12x^2 - 21x + 10$ — 3 variations = 3 or 1 negative real root

$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1}$ — The graph shows x-intercepts at -10 and -1 where there is tangency.

test:

-10	1	12	21	10
		-10	-20	-10
	1	2	1	0

-1	1	12	21	10
		-1	-11	-10
	1	11	10	0

Now test with the previous quotient to look for the repeating root.

-1	1	11	10
		-1	-10
	1	10	0

$x = \{-1, -1, -10\}$

4) $(x - 4)(x^2 + 4x + 16)$

5) $(x + 3)(x^2 - 3x + 9)$

6) $(x + 6)(x^2 - 6x + 36)$