Section 1.6

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Transformations of Functions

- How does the graph of a function change if you add a constant to the independent variable? To the dependent variable?
- How does the graph of a function change if you multiply a constant times the independent variable? To the dependent variable?
- How does the graph of a function change if you change the sign of the independent variable? Of the dependent variable?

Topics

- Vertical shifts
- Horizontal shifts
- Reflection about the x-axis
- Reflection about the y-axis
- Vertical stretching and shrinking
- Horizontal stretching and shrinking

Vertical Shifts

Given the graph

$$y = f(x)$$

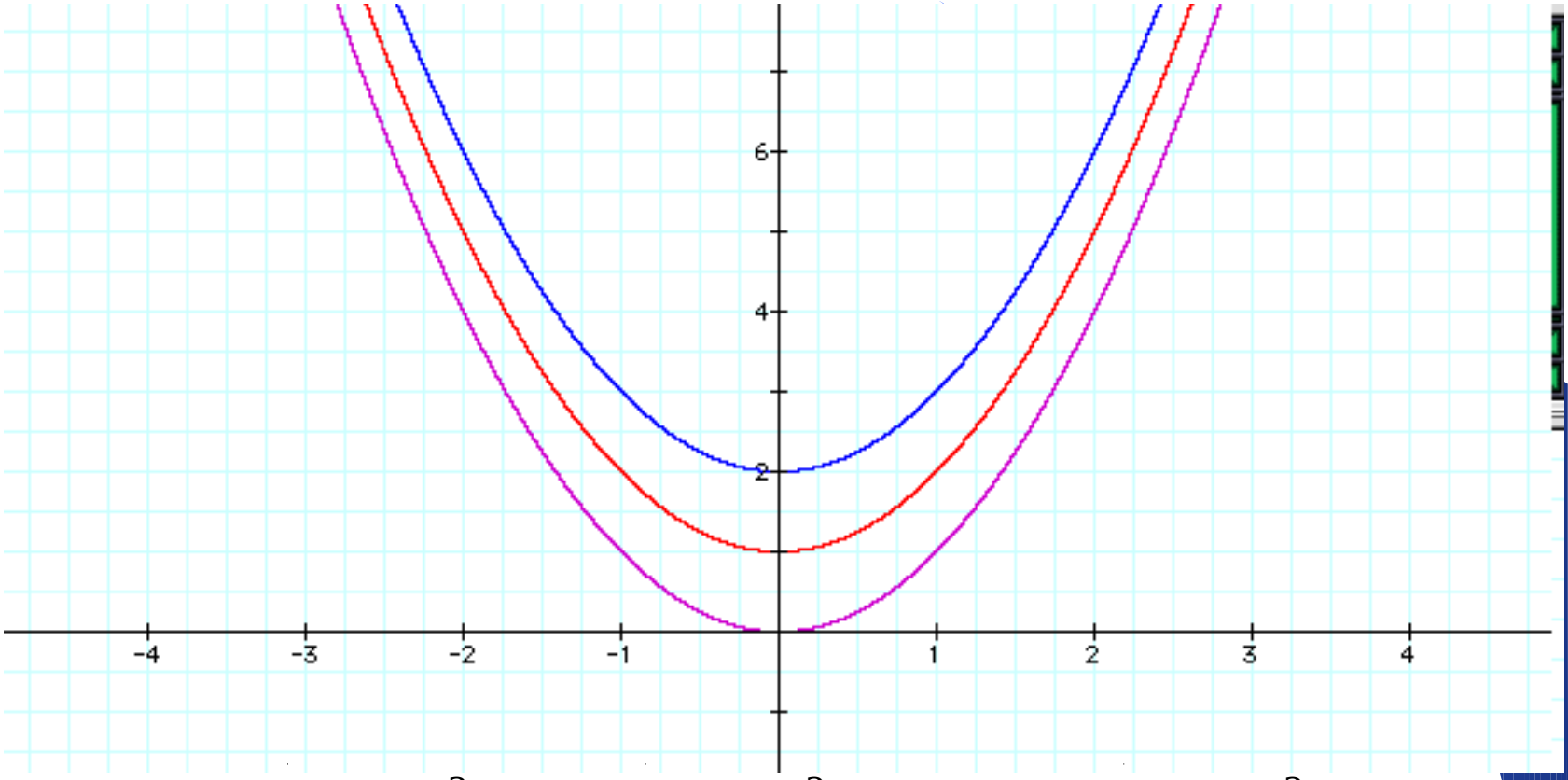
you get the graph

$$y = f(x) + k$$

by moving the first graph up k units (for $k>0$)

Adding a constant outside the function moves the graph of the function **up**.

Vertical Shifting



$y = x^2$ $y = x^2 + 1$ $y = x^2 + 2$

Vertical Shifting (Continued)

Given the graph

$$y = f(x)$$

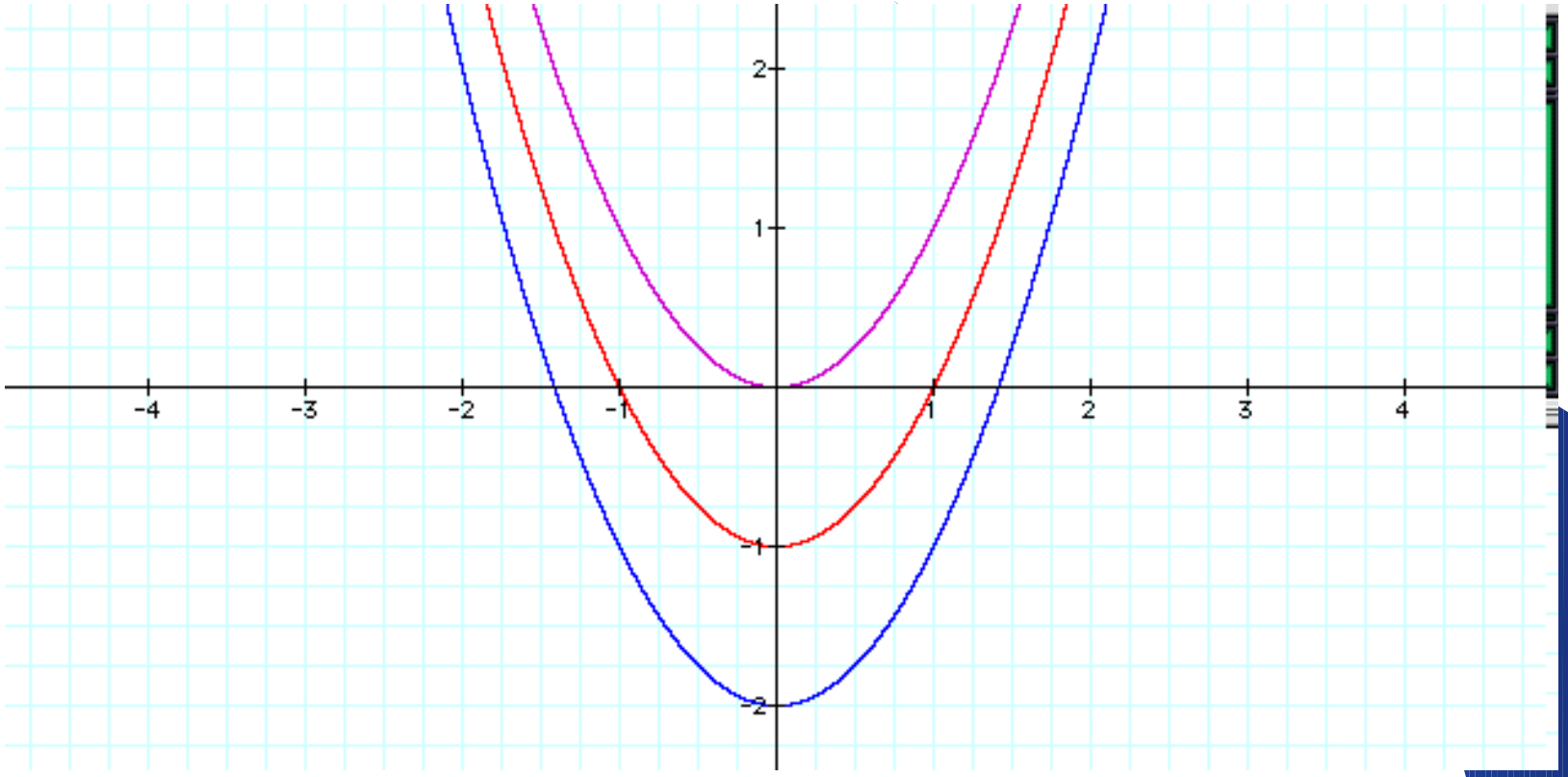
you get the graph

$$y = f(x) - k$$

by moving the first graph down k units (for $k>0$)

Subtracting a constant outside the function moves the graph of the function **down**.

Vertical Shifting (Continued)



$y = x^2$ $y = x^2 - 1$ $y = x^2 - 2$

Horizontal Shifting

Given the graph

$$y = f(x)$$

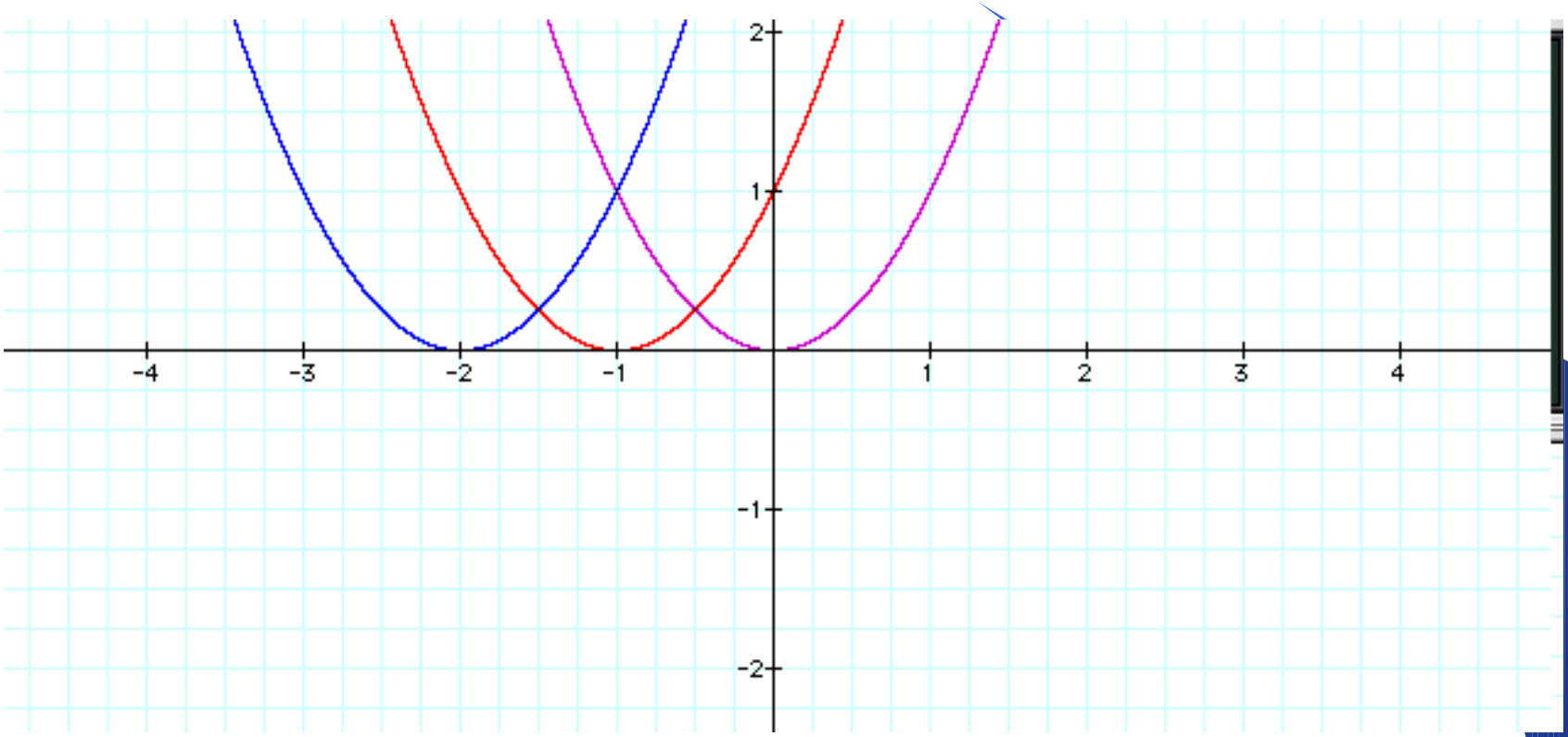
you get the graph

$$y = f(x + k)$$

by moving the first graph left k units (for $k>0$)

Adding a constant inside the function moves the graph of the function **left**.

Horizontal Shifting



$y = x^2$ $y = (x+1)^2$ $y = (x+2)^2$

Horizontal Shifting (Continued)

Given the graph

$$y = f(x)$$

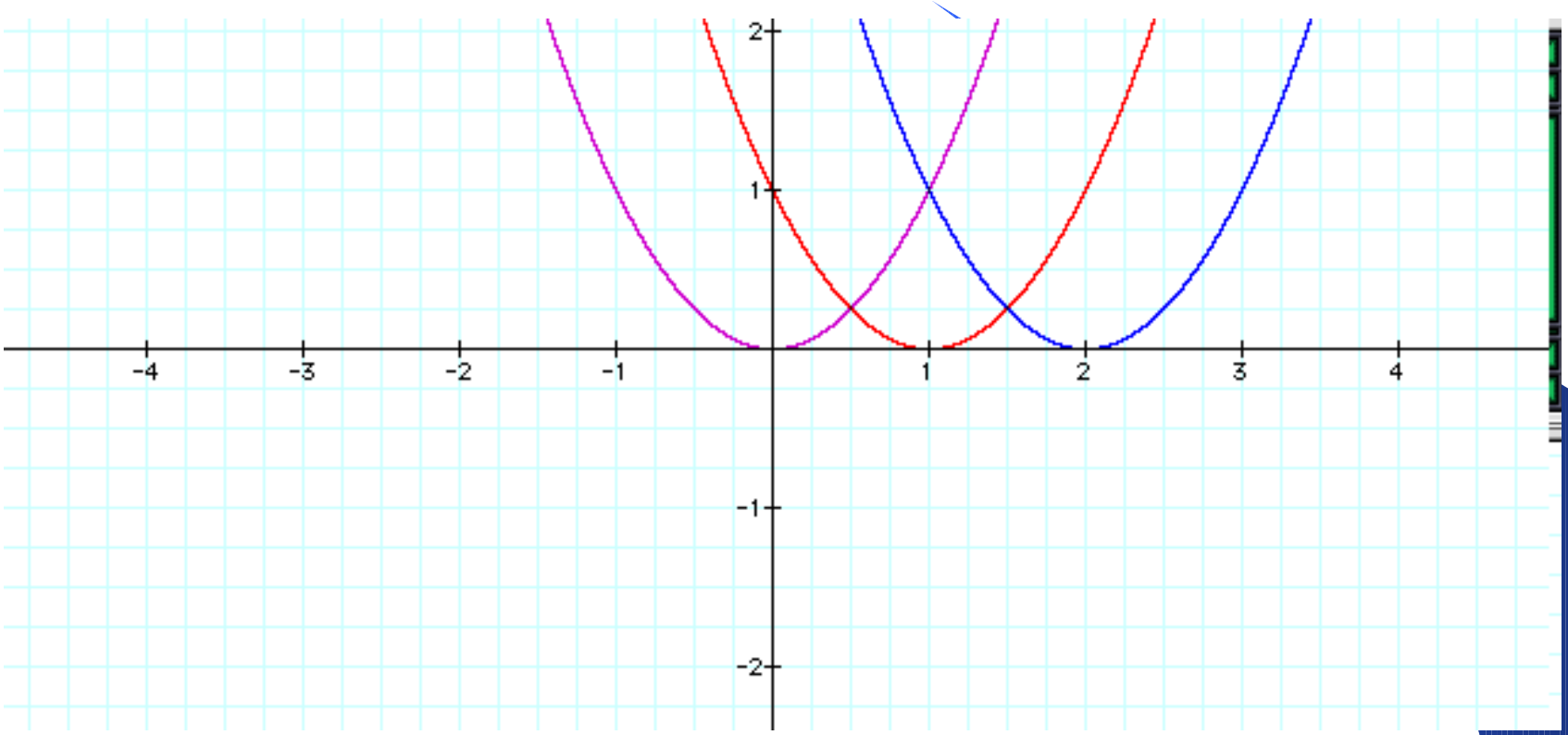
you get the graph

$$y = f(x - k)$$

by moving the first graph right k units (for $k>0$)

Subtracting a constant inside the function moves the graph of the function **right**.

Horizontal Shifting (Continued)



$y = x^2$ $y = (x - 1)^2$ $y = (x - 2)^2$

Reflections about the *x*-axis

Given the graph

$$y = f(x)$$

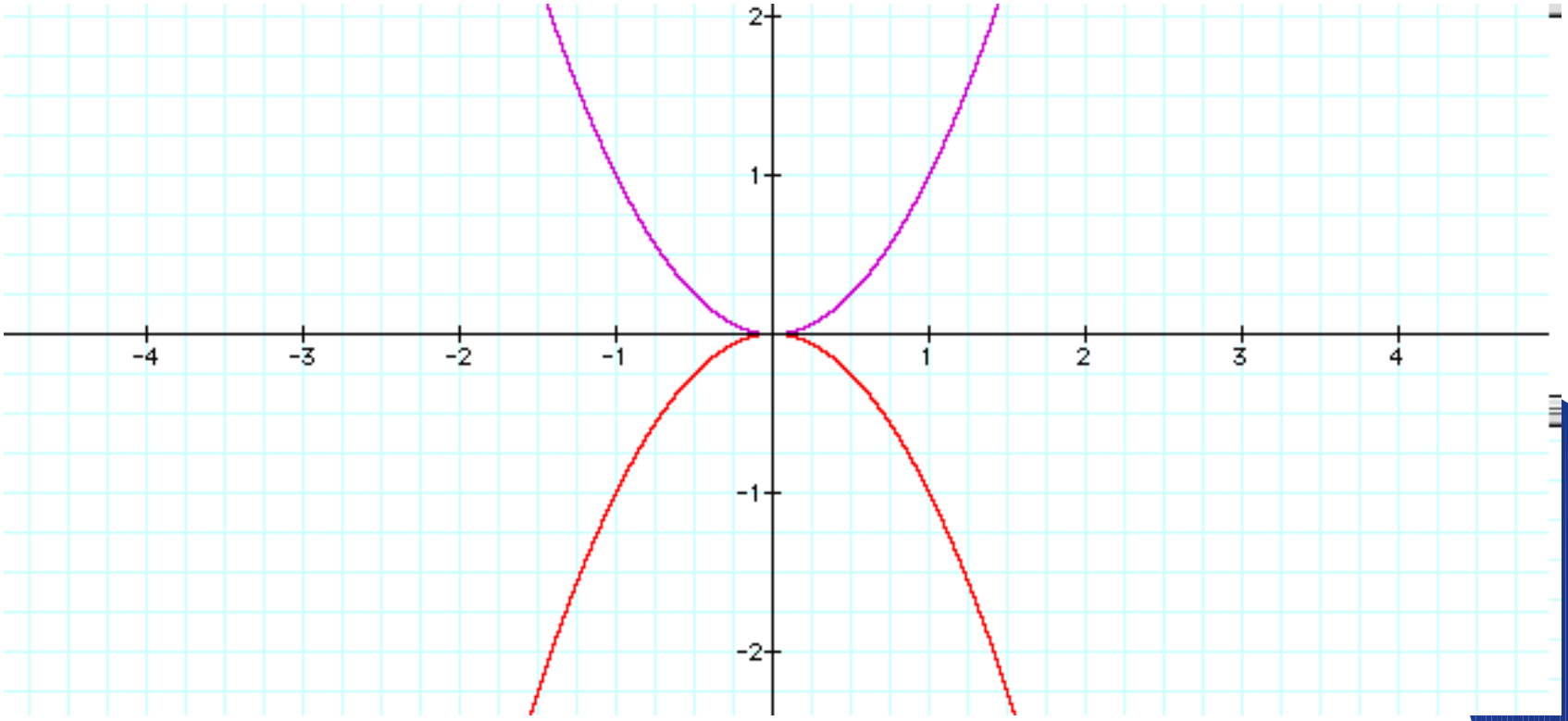
you get the graph

$$y = -f(x)$$

by reflecting the first graph across the *x*-axis.

Changing y to $-y$ reflects the graph across the *x*-axis.

Reflections about the *x*-axis



$y = x^2$ $y = - x^2$

Reflections about the *y*-axis

Given the graph

$$y = f(x)$$

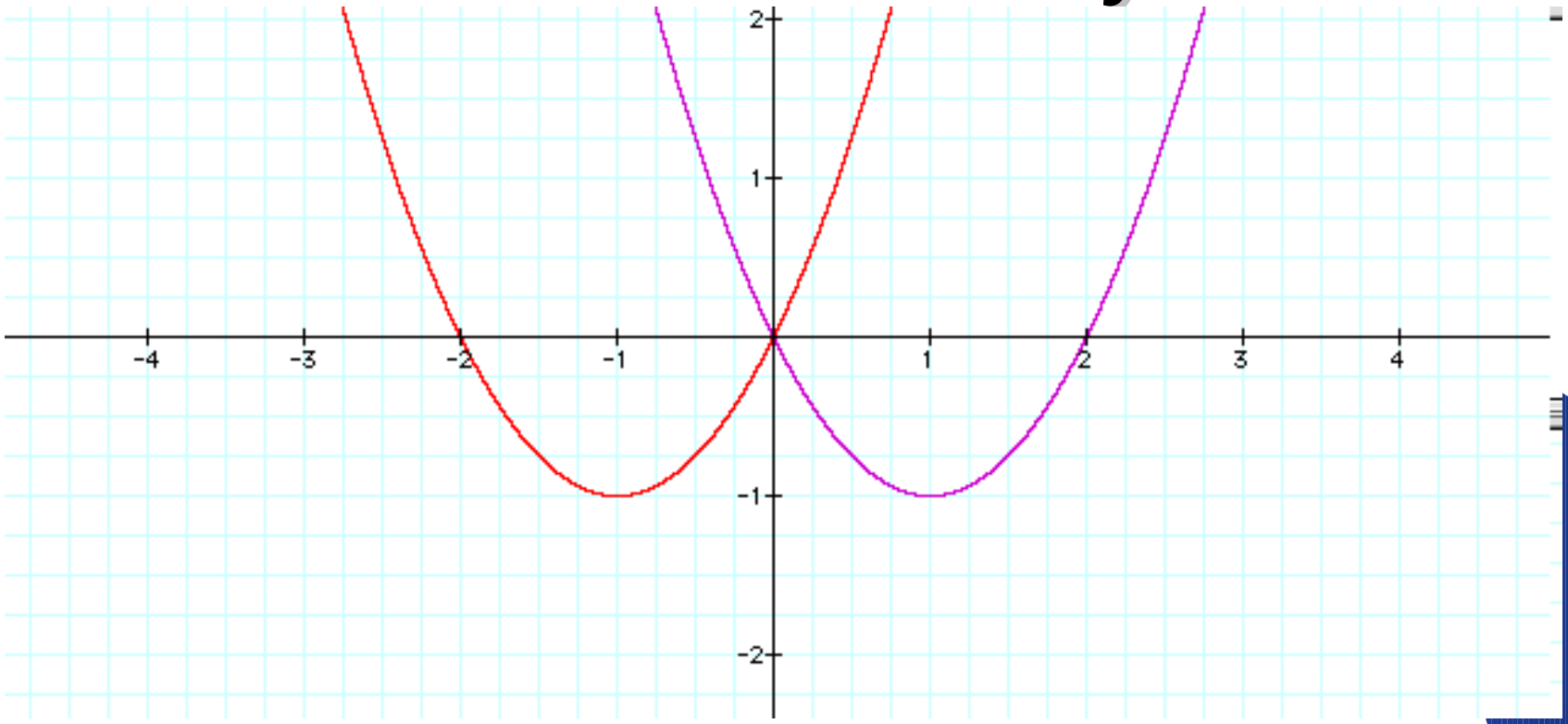
you get the graph

$$y = f(-x)$$

by reflecting the first graph across the *y*-axis.

Changing x to $-x$ reflects the graph across the *y*-axis.

Reflections about the *y*-axis



$y = x^2 - 2x$ $y = (-x)^2 - 2(-x) = x^2 + 2x$

Vertical Stretching

Given the graph

$$y = f(x)$$

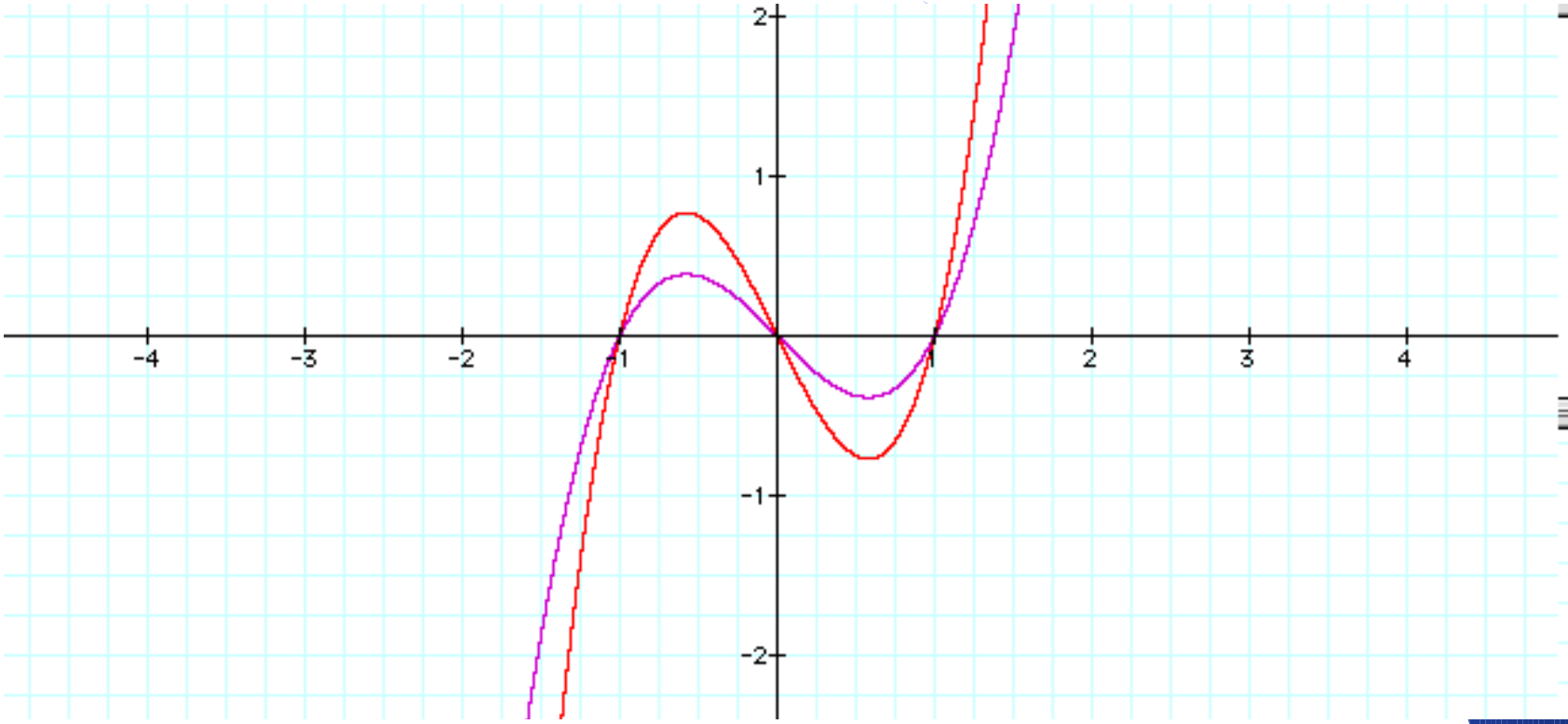
you get the graph

$$y = kf(x) \text{ for } k > 1$$

by stretching the first graph vertically by a factor of k.

Multiplying y by $k>1$ stretches the graph vertically by a factor of k.

Vertical Stretching



$y = x^3 - x$ $\qquad$ $y = 2(x^3 - x)$

Vertical Shrinking

Given the graph

$$y = f(x)$$

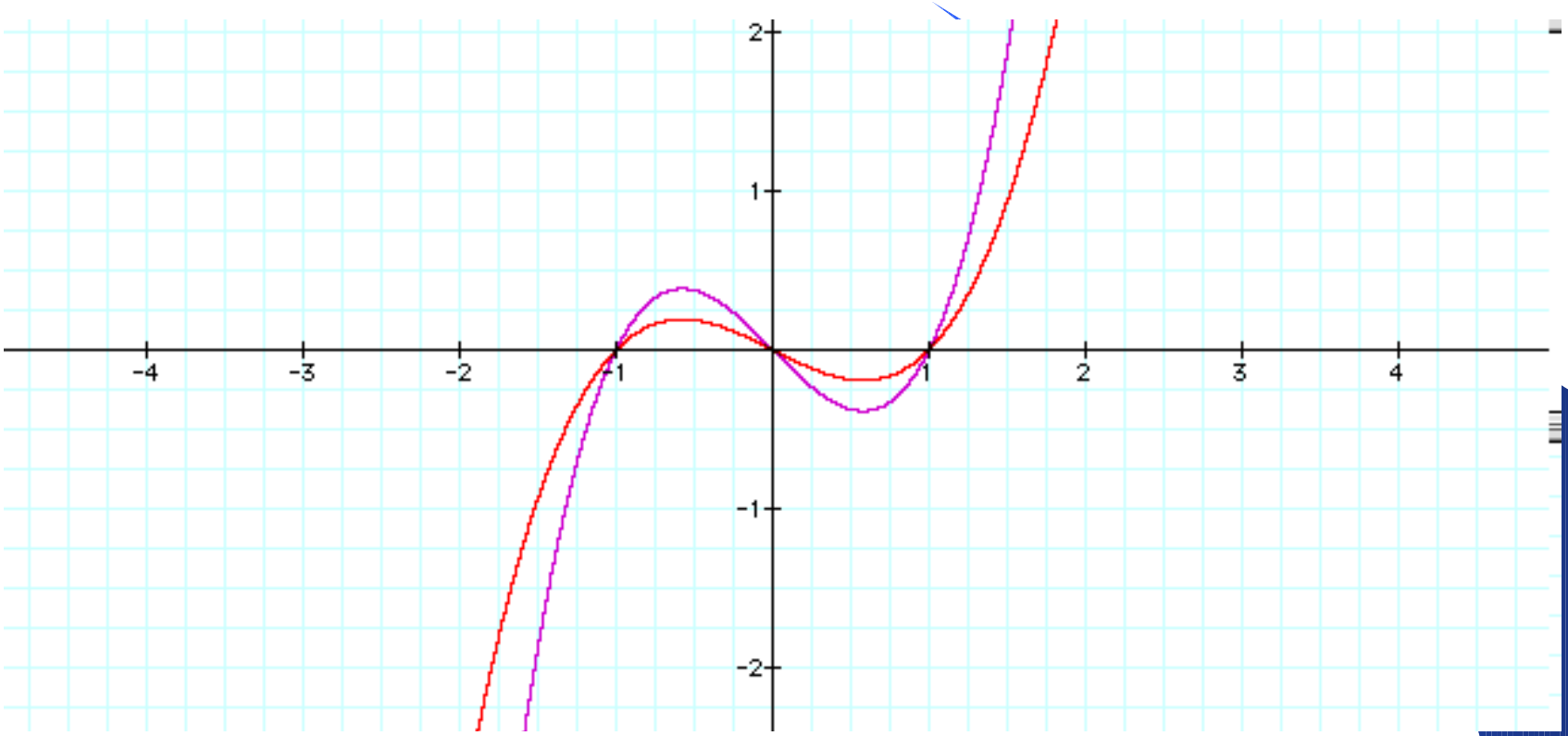
you get the graph

$$y = kf(x) \text{ for } 0 < k < 1$$

by shrinking the first graph vertically by a factor of k.

Multiplying y by $0<k<1$ shrinks the graph vertically by a factor of k.

Vertical Shrinking



$y = x^3 - x$ $\qquad$ $y = \frac{1}{2}(x^3 - x)$

Horizontal Stretching

Given the graph

$$y = f(x)$$

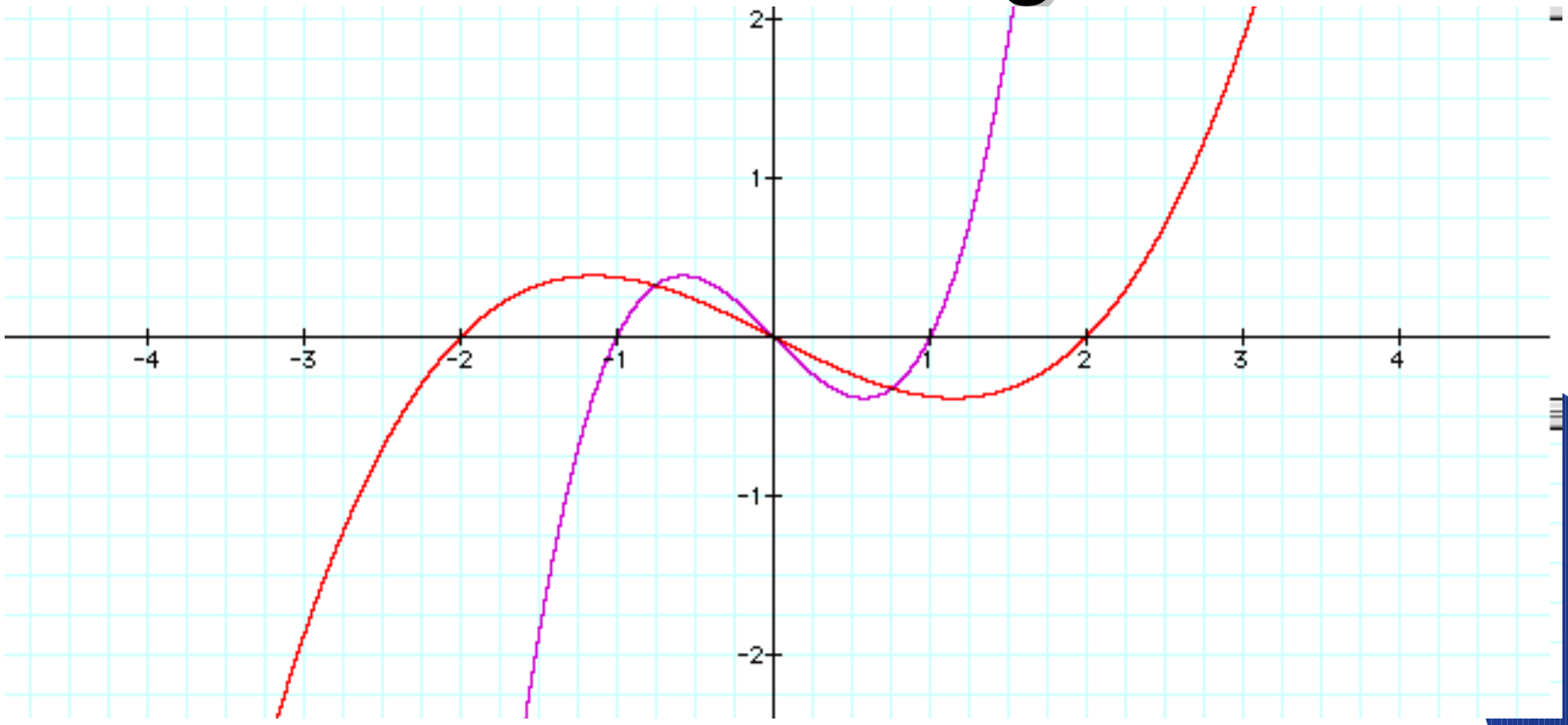
you get the graph

$$y = f(kx) \text{ for } 0 < k < 1$$

by stretching the first graph horizontally by a factor of k.

Multiplying x by $0<k<1$ stretches the graph horizontally by a factor of k.

Horizontal Stretching



$y = x^3 - x$ $\qquad$ $y = ((\frac{1}{2}x)^3 - (\frac{1}{2}x))$

Horizontal Shrinking

Given the graph

$$y = f(x)$$

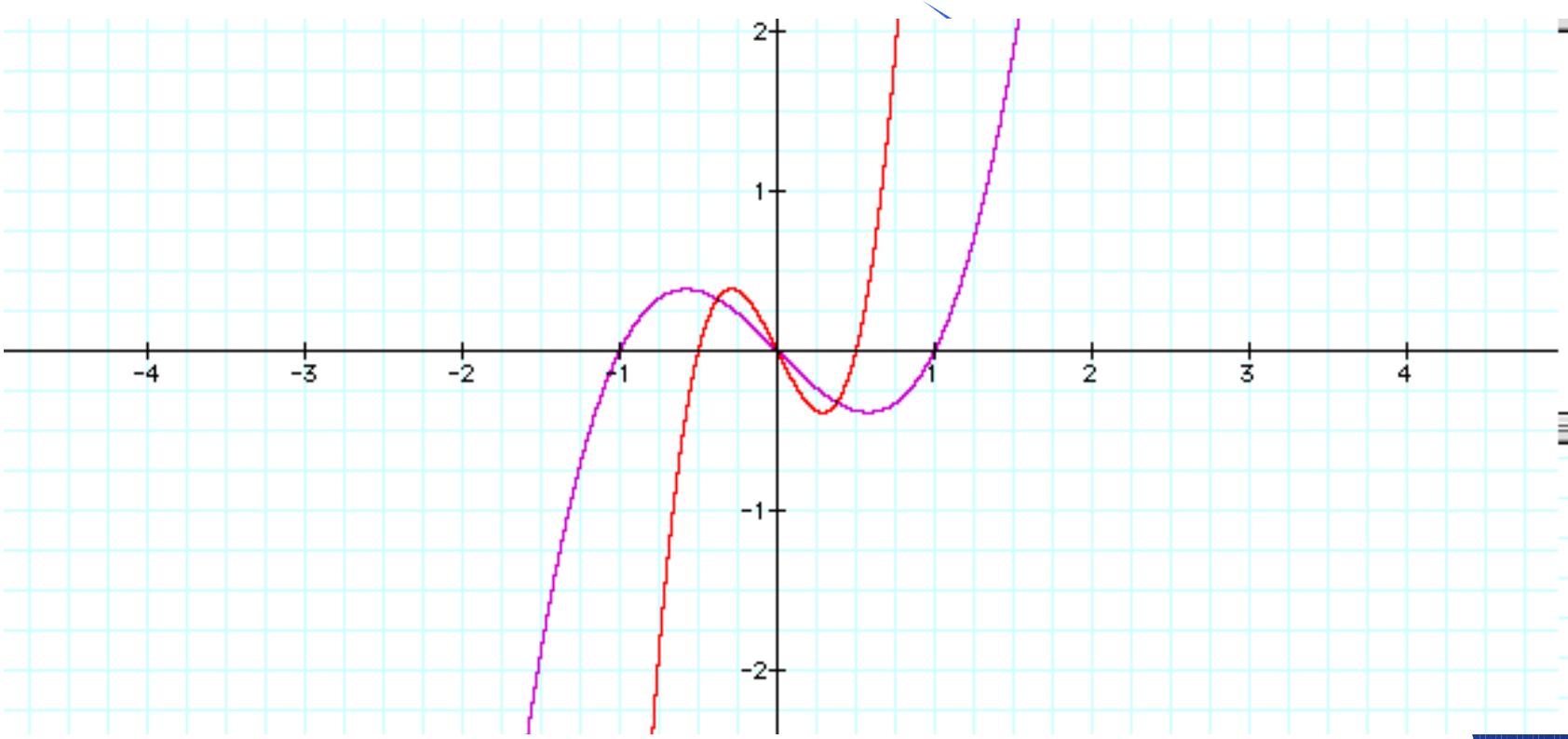
you get the graph

$$y = f(kx) \text{ for } k > 1$$

by shrinking the first graph horizontally by a factor of k.

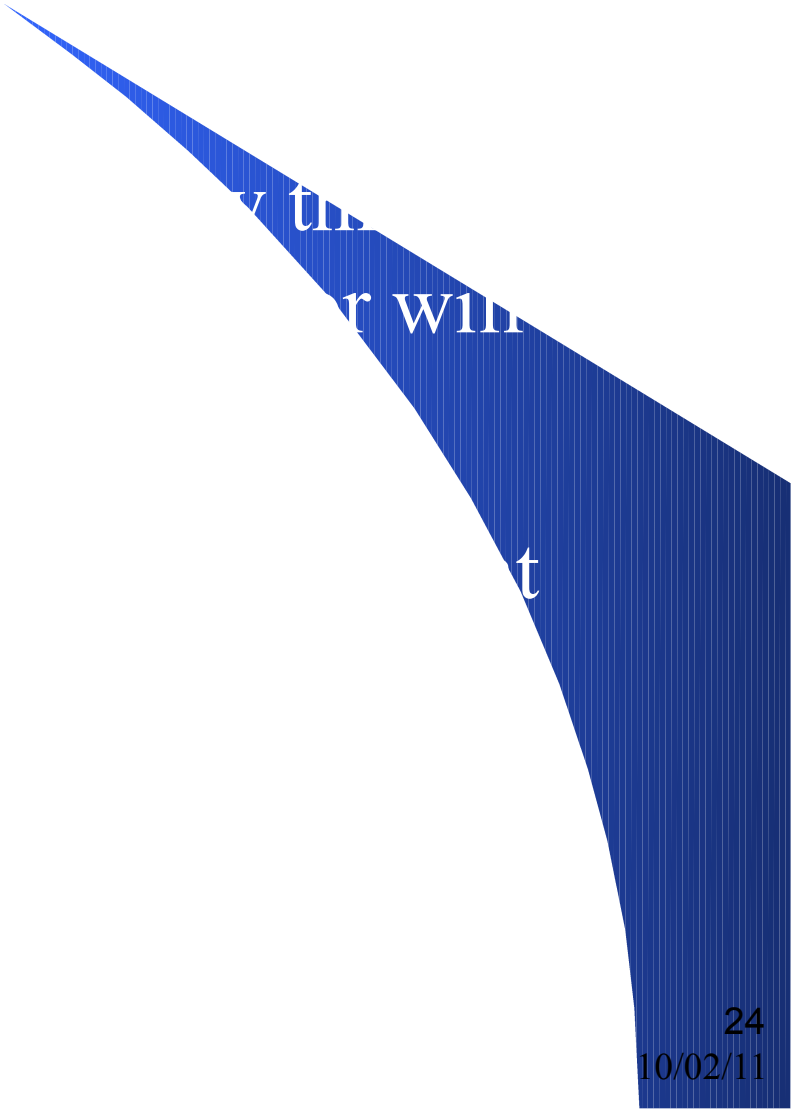
Multiplying x by $k>1$ shrinks the graph horizontally by a factor of k.

Horizontal Shrinking



$y = x^3 - x$ $\qquad$ $y = ((2x)^3 - (2x))$

Why Are We Learning This?



Summary

- We have learned about vertical and horizontal shifting
- We have learned about vertical and horizontal stretching and shrinking
- We have learned about reflections about both axes

Things To Do

- Reread Section 1.6
- Do the homework for Section 1.6
- Read Section 1.7

One More Thing . . .

Have a nice day