MCA Test 8 Review Answer key

1, Solve for the interval [0, 360]

Identity substitution

$\sin 2x = 2\sin x\cos x$

$\csc = \frac{1}{\sin x}$

$\cos^2 - \sin 2x \csc x = 3$

$\cos^2 - \cancel{2\sin x}\cos x \frac{1}{\cancel{\sin x}} - 3 = 0$

$\cos^2 x - 2\cos x - 3 = 0$

$(\cos x - 3)(\cos x + 1) = 0$

$\cos x = 3$ reject | $\cos x = -1$

$\cos^{-1} x = -1$

$\boxed{\cos = 180}$

2. Find all asymptotes + holes:

$f(x) = \frac{x^2 + x - 30}{x^2 - 5x}$ $\quad \frac{(x+5)(x+6)}{(x-5)\,x}$

vertical asymptote = $x = 0$

horizontal asymptote - $y = 1$

hole at $(5, 11/5)$

3, Solve: $3^x + 5 = 30$

$3^x = 25$

$x \log 3 = \log 25$

$x = \frac{\log 25}{\log 3}$

$= 2.93$

4. Solve

$\log(3x) - \log(x+2) = \log 5$

$\log \frac{3x}{x+2} = \log 5$ $\quad \frac{3x}{x+2} = 5$

$3x = 5(x+2) \rightarrow 3x = 5x + 10 \Rightarrow 3x - 5x = 10$

$-2x = 10$

$x = -5$

$\{\ \}$ Reject -5

Remember:

$y = r\cos\theta$

$x = r\sin\theta$

$r = \sqrt{\sin^2 + \cos^2}$

$\theta = \frac{\sin}{\cos} = \tan$

5

$z = 12(\cos 300° + i \sin 300)$

$12\left(\frac{1}{2} + i - \frac{\sqrt{3}}{2}\right)$

$\underline{6 + 6\sqrt{3}i}$

6. $z = 3 - 7i$

$r = \sqrt{3^2 + (-7)^2} = \sqrt{9+49} = \sqrt{58}$

$\tan\theta = \frac{-7}{3} = \theta\ 293$ (Quad 3)

$\sqrt{58}(\cos 293 + i \sin 293)$

7. De Moivre's Theorem

$r^n(\cos n\theta + i \sin n\theta)$

$(-1 + \sqrt{3}i)^8$ $\quad r = \sqrt{(-1)^2 + (\sqrt{3})}$

$\tan \frac{\sqrt{3}}{1} = -\sqrt{3}$ $\quad r = \sqrt{1+3} = \sqrt{4} = 2$

sin + | cos

cos − |

$2^8(\cos 120(8) + i \sin(120 \cdot 8)$

$256(\cos 960 + i \sin 960°)$

$256(\cos 240 + i \sin 240)$

$256(-\cos 60 + i - \sin 60$

$256\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}\right)$

$125 - 128i\sqrt{3}$

8) $z_1 z_2 = r_1 r_2(\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)$

$\frac{z_1}{z_2} = \left(\frac{r_1}{r_2}\right)[\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)$

$\frac{26(\cos 68 + i \sin 68)}{13(\cos 27 + i \sin 27)} = 2(\cos 68 - 27) + i \sin(68 - 27)$

$= 2(\cos 41 + i \sin 41)$

9) Find all the square roots of $16(\cos 120° + i \sin 120°)$ in $a+bi$ form

$\sqrt[n]{r}(\cos \frac{120 + 360(0)}{2} + i \sin(\frac{120 + 360(0)}{2})$

$2\sqrt{16}(\cos 60 + i \sin 60) = 4\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$ $2 + 2\sqrt{3}i$

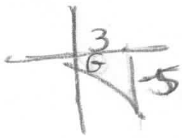
$4(\cos(\frac{120 + 360}{2}) + i \sin(\frac{120 + 360}{2}) = 4[\cos \frac{480}{2}) + i \sin \frac{480}{2}$

$4(\cos(240) + i \sin 240 = 4\left(-\frac{1}{2} + i - \frac{\sqrt{3}}{2}\right) = -2 + 2\sqrt{3}i$

MCA Test Review Ans Key pg 2

Convert $(3,-5)$ to Polar Form

$r=\sqrt{3^2+(-5)^2}=\sqrt{9+25}=\sqrt{34}$



$\tan\theta=\frac{-5}{3}$

$\tan^{-1}\left(\frac{-5}{3}\right)=301^\circ$

$(\sqrt{34},301^\circ)$ polar point

Convert $(8,225^\circ)$ to x,y

$x=r\cos\theta$ $\quad 8\cos 225 \left(8\left(-\frac{\sqrt{2}}{2}\right)\right)$

$y=r\sin\theta$ $\quad 8(\sin 225)\, 8\left(-\frac{\sqrt{2}}{2}\right)$

$(-4\sqrt{2},-4\sqrt{2})=(x,y)$

a) $r=3\cos 17\theta$ will have 17 loops

b) $r=5\sin 14\theta$ will have 28 loops