

Homework #92

Answers

From Houghton-Mifflin Precalculus

3rd Edition

p789:

20) $u = i + 2j, v = i - 3k$

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 1 & 0 & -3 \end{vmatrix} = (-6 - 0)i - (-3 - 0)j + (0 - 2)k = -6i + 3j - 2k$$

$$\|u \times v\| = \sqrt{36 + 9 + 4} = 7; \quad \frac{u \times v}{\|u \times v\|} = (-6/7)i + (3/7)j - (2/7)k$$

21) $u = -2i + j + 3k, v = i + 4j + 6k$

$$u \times v = \begin{vmatrix} i & j & k \\ -2 & 1 & 3 \\ 1 & 4 & 6 \end{vmatrix} = (6 - 12)i - (-12 - 3)j + (-8 - 1)k = -6i + 15j - 9k$$

$$\|u \times v\| = \sqrt{36 + 225 + 81} = \sqrt{342} = 3\sqrt{38}; \quad \frac{u \times v}{\|u \times v\|} = (-2/\sqrt{38})i + (5/\sqrt{38})j - (3/\sqrt{38})k$$

27) $u = 3i + 4j + 6k, v = 2i - j + 5k$

$$u \times v = \begin{vmatrix} i & j & k \\ 3 & 4 & 6 \\ 2 & -1 & 5 \end{vmatrix} = (20 - 6)i - (15 - 12)j + (-3 - 8)k = 26i - 3j - 11k$$

$$\text{Area} = \|u \times v\| = \sqrt{676 + 9 + 121} = \sqrt{806} = 28.39$$

31) $A(2, -1, 4), B(3, 1, 2), C(0, 5, 6), D(-1, 3, 8)$

$$AB = \langle 1, 2, -2 \rangle = i + 2j - 2k \quad BC = \langle -3, 4, 4 \rangle = -3i + 4j + 4k$$

$$CD = \langle -1, -2, 2 \rangle = -i - 2j + 2k \quad DA = \langle 3, -4, -4 \rangle = 3i - 4j - 4k$$

Since pairs of opposite sides are vectors that are scalar multiples they are parallel making ABCD a parallelogram.

$$AB \times BC = \begin{vmatrix} i & j & k \\ 1 & 2 & -2 \\ -3 & 4 & 4 \end{vmatrix} = (8 - -8)i - (4 - 6)j + (4 - -6)k = 16i + 2j + 10k$$

$$\text{Area} = \|AB \times BC\| = \sqrt{256 + 4 + 100} = \sqrt{360} = 6\sqrt{10}$$

$$\sin \theta = \frac{\|AB \times BC\|}{\|AB\| \|BC\|} = \frac{6\sqrt{10}}{(\sqrt{1 + 4 + 4})(\sqrt{9 + 16 + 16})} = \frac{6\sqrt{10}}{3\sqrt{41}} = .9877$$

$$\theta = 81.02. \text{ Not orthogonal}$$

34) $A(1, -4, 3), B(2, 0, 2), C(-2, 2, 0)$

$$AB = \langle 1, 4, -1 \rangle = i + 4j - k$$

$$BC = \langle -4, 2, -2 \rangle = -4i + 2j - 2k$$

$$CA = \langle 3, -6, 3 \rangle = 3i - 6j + 3k$$

$$AB \times BC = \begin{vmatrix} i & j & k \\ 1 & 4 & -1 \\ -4 & 2 & -2 \end{vmatrix} = (-8 - -2)i - (-2 - 4)j + (2 - -16)k = -6i + 6j + 18k$$

$$\text{Area} = \frac{1}{2}\|AB \times BC\| = \frac{1}{2}\sqrt{36 + 36 + 324} = \frac{1}{2}\sqrt{396} = 3\sqrt{11}$$