Aim: Aim: How do we solve polynomial equations using the n^{th} roots formula?

Do Now: 1) Solve: $x^4 + 16 = 0$

Development: >How can we do the "Do Now"?
<We can try to solve algebraically but as this is not factorable that could be somewhat problematic.

We start with $x^4 + 16 = 0$ then $x^4 = -16$. This means we're trying to find $\sqrt[4]{-16 + 0i}$ This means that $r = \sqrt{(16)^2 + (0)^2} = 16$ and $\tan\theta = 0/16 = 0$ so $\theta = 0^\circ$ or since we're working with -16 actually $\theta = 180°$.

So we have $\sqrt[4]{16}\left(\cos\frac{180 + 360k}{4} + i\sin\frac{180 + 360k}{4}\right)$

And using k = 0, 1, 2, 3 we get:

$$2\left(\cos\frac{180+360(0)}{4} + i\sin\frac{180+360(0)}{4}\right) = 2\left(\cos\frac{180}{4} + i\sin\frac{180}{4}\right) =$$
$$2(\cos 45 + i\sin 45) = 2\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}i}{2}\right) = \sqrt{2} + \sqrt{2}i$$

$$2\left(\cos\frac{180+360(1)}{4} + i\sin\frac{180+360(1)}{4}\right) = 2\left(\cos\frac{540}{4} + i\sin\frac{540}{4}\right) =$$
$$2(\cos 135 + i\sin 135) = 2\left(\frac{-\sqrt{2}}{2} + \frac{\sqrt{2}i}{2}\right) = -\sqrt{2} + \sqrt{2}i$$

$$2\left(\cos\frac{180+360(2)}{4} + i\sin\frac{180+360(2)}{4}\right) = 2\left(\cos\frac{900}{4} + i\sin\frac{900}{4}\right) =$$
$$2(\cos 225 + i\sin 225) = 2\left(\frac{-\sqrt{2}}{2} - \frac{\sqrt{2}i}{2}\right) = -\sqrt{2} - \sqrt{2}i$$

$$2\left(\cos\frac{180+360(3)}{4} + i\sin\frac{180+360(3)}{4}\right) = 2\left(\cos\frac{1260}{4} + i\sin\frac{1260}{4}\right) =$$
$$2(\cos 315 + i\sin 315) = 2\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}i}{2}\right) = \sqrt{2} - \sqrt{2}i$$

Application:
2) Solve: $x^3 + 64i = 0$

Answer:
2) $x^3 = -64i$ $r = \sqrt{0 + (-64)^2} = 64$, $\tan\theta = -64/0$, $\theta = 270°$

$x = \sqrt[3]{64}(\cos(270/3) + i\sin(270/3)) = 4(\cos 90 + i\sin 90) = 4(0 + 1i) = 4i$

$x = \sqrt[3]{64}(\cos(630/3) + i\sin(630/3)) = 4(\cos 210 + i\sin 210)$
$= 4(-\sqrt{3}/2) + 4(-\tfrac{1}{2}i) = -2\sqrt{3} - 2i$

$x = \sqrt[3]{64}(\cos(990/3) + i\sin(990/3)) = 4(\cos 330 + i\sin 330)$
$= 4(\sqrt{3}/2) + 4(-\tfrac{1}{2}i) = 2\sqrt{3} - 2i$

Homework: HEATH: p519: #86-88, 91
3rd ed. HOUGHTON-MIFFLIN: p477: #108-110, 113