Pre-Calculus - Honors Lesson 19

Aim: How do we solve and graph inequalities?

Do Now: Solve for x:

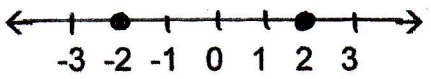
1) $|x| + 3 = 5$ 2) $|x| + 3 < 5$

Development: $|x| = 2$

$x = \{-2, 2\}$

If we solve an absolute value equation we see that we have two solutions and as seen on a # line we have two points that satisfy the solution set.

Graph:



If we solve $|x| + 3 < 5$

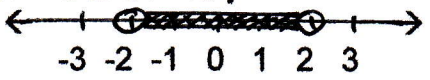
$|x| < 2$

>How can we write this without the "| |"?

<We see that if $x < 2$ then $x > -2$. As the sign of the number changes so must the direction of the inequality - just like in a regular inequality where we change the sign when we divide by a negative.

So we have a result of $\{x \mid x > -2 \text{ and } x < 2\}$

The graph of this will be:



How else might we write this CONJUNCTION?

$\{x \mid -2 < x < 2\}$

So the rule is: If $|x| < k$, where k is positive, its solution set is

$\{x \mid -k < x < k\}$ or $\{x \mid (x > -k) \wedge (x < k)\}$

>Suppose we consider $|x| > 2$. How can we write this without the "| |"?

<We see that $\{x \mid x < -2 \text{ or } x > 2\}$ - here, in order to get the correct values we use a DISJUNCTION - the solutions exist but with NO elements in common. The graph of this will be:

So the rule is: If $|x| > k$, where k is positive, its solution set is

$\{x \mid x < -k \text{ or } x > k\}$ or $\{x \mid (x < -k) \vee (x > k)\}$

ex 3) Find the solution set:

$|2x + 3| < 7$ Write the given inequality.

$2x + 3 < 7$ $\quad$ $2x + 3 > -7$ Write the derived equation as 2 separate equations.

$-3 \quad -3$ $\quad$ $-3 \quad -3$

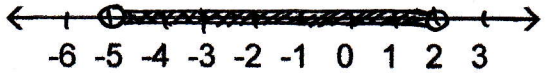
$2x < 4$ $\quad$ $2x > -10$ Solve the inequality.

$x < 2$ $\quad$ $x > -5$

So $(x > -5) \wedge (x < 2)$ State the solution set.

or $-5 < x < 2$

Graph:



ex 4) Find the solution set:

$|3 + y| - 2 \geq 0$

$+2 \quad +2$

$|3 + y| \geq 2$

$3 + y \geq 2$ $\quad$ $3 + y \leq -2$

$-3 \quad -3$ $\quad$ $-3 \quad -3$

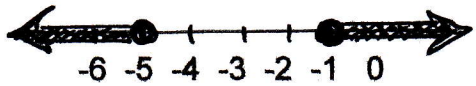
$y \geq -1$ $\quad$ $y \leq -5$

$(y \leq -5) \vee (y \geq -1)$

Note: There is NO need to check inequalities as the answers present a set of values. The answers do not need to fit exactly and often do not.

Note: There is no alternate way to write a disjunction. A form that would be equivalent to the interval form that we use for conjunctions does not exist.

Graph:



5) Solve $x^2 - 2x - 3 < 0$.

>How can we solve this?

<Solve as an equation first and then use the same rules as those for absolute value inequalities.

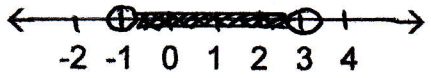
$(x - 3)(x + 1) = 0$

$x - 3 = 0 \quad x + 1 = 0$

$x = \{3, -1\}$

The solution set for the inequality is $-1 < x < 3$

Graph:



Applications: Find the solution set for:

6) $|x - 20| \leq 4$ 7) $|x - 7| < 6$ 8) $x^2 + 4x + 4 \geq 9$ 9) $x^2 - 6x + 9 < 16$

Answers:

6) $x - 20 \geq -4 \quad x - 20 \leq 4$

$x \geq 16 \quad x \leq 24$

$16 \leq x \leq 24$

7) $x - 7 > -6 \quad x - 7 < 6$

$x > 1 \quad x < 13$

$1 < x < 13$

8) $x^2 + 4x + 4 = 9$

$x^2 + 4x - 5 = 0$

$(x + 5)(x - 1) = 0$

$x = \{-5, 1\}$

$(x \leq -5) \vee (x \geq 1)$

9) $x^2 - 6x + 9 = 16$

$x^2 - 6x - 7 = 0$

$(x - 7)(x + 1) = 0$

$x = \{7, -1\}$

$-1 < x < 7$

Homework: HEATH: p192-193 #29, 30, 32, 44, 54, 59, 60 p165 #24, 38, 43, 48

HOUGHTON-MIFFLIN: pA73 #32-34, 36, 54, 59, 60 p137: #18, 32, 35, 50