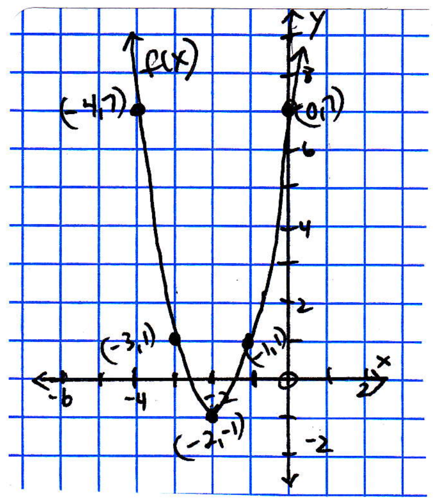
Pre-Calculus - Honors | Lesson 18

Aim: How does the graph of a quadratic equation relate to the standard form?

Do Now: 1) Graph: $f(x) = 2x^2 + 8x + 7$ over $-4 \leq x \leq 0$



Development: We see in #1 we have a fairly simple quadratic function.

>What is the turning point of this graph?

<The turning point is (-2,-1).

The turning point of a parabola is also known as the vertex. We sometimes refer to this vertex with the letters (h, k) - like the center of a circle.

We're used to quadratic equations appearing in the format $f(x) = ax^2 + bx + c$ where we can "solve" when $f(x) = 0$ and where the graphed parabola opens up when $a > 0$ and opens down when $a < 0$ and a can never = 0. This is called the general form.

The quadratic function also has another form that is commonly known as the standard form of a quadratic function.

This is $f(x) = a(x - h)^2 + k$, a not = 0. The (h, k) is the turning point and a's sign represents what it always does. The meaning of a is not always apparent but relates to a distance from the vertex to other significant parts of the graph. We'll learn more about that later.

The big question is how do we convert the equation from one form to another?

$f(x) = 2x^2 + 8x + 7$

$= 2(x^2 + 4x) + 7$ 1) Factor 2 out of x terms only.

$= 2(x^2 + 4x + 4) + 7 - 2(4)$ 2) Complete the square and account for the added value (including the outside multiple by subtracting at the back.

$= 2(x + 2)^2 - 1$ 3) Simplify and express the trinomial as a binomial squared.

This now gives us the format we are looking for.

From this we can see that based on the standard form (h, k), the turning point, is (-2,-1) and knowing a is positive means that the parabola opens up.

Note: This is VERY closely related to another form of the quadratic equation known as the vertex form. It looks like this: $y - k = 4p(x - h)^2$.
This form is not in function notation but it does allow us to see what is really happening with k.

Now this is not enough information to graph the parabola but substituting 0 for x and y can give us the y and x intercepts, respectively. With all this information we can graph the parabola without using a table.

We can use this formula for finding an equation as well:

2) Find an equation for the parabola that has its vertex at (1,2) and that passes through the point (0,0).

We start by substituting the values for (h,k): $f(x) = a(x - h)^2 + k$

$f(x) = a(x - 1)^2 + 2$

We can substitute in (0,0) for x and y: $0 = a(0 - 1)^2 + 2$

Then we can simplify: $0 = a(-1)^2 + 2$

$0 = 1a + 2$

$-2 = a$

So the final equation is $f(x) = -2(x - 1)^2 + 2$ in standard form, or to find in general form, simplify: $f(x) = -2(x^2 - 2x + 1) + 2$

$f(x) = -2x^2 + 4x - 2 + 2$

$f(x) = -2x^2 + 4x$

Applications:

3) Write the equation in standard form: $f(x) = -x^2 + 6x - 8$

$$f(x) = -x^2 + 6x - 8$$
$$= -(x^2 - 6x) - 8$$
$$= -(x^2 - 6x + 9 - 9) - 8$$
$$= -(x^2 - 6x + 9) - (-9) - 8$$

$f(x) = -(x - 3)^2 + 1$

From this we see that the vertex is (3,1) and opens downward.

4) Find the quadratic function that has a vertex of (3,4) and the point (1,2).

$$f(x) = a(x - 3)^2 + 4$$
$$2 = a(1 - 3)^2 + 4$$
$$2 = a(-2)^2 + 4$$
$$-2 = 4a$$
$$-\tfrac{1}{2} = a$$

$f(x) = -\tfrac{1}{2}(x - 3)^2 + 4$

Homework: Worksheet #18