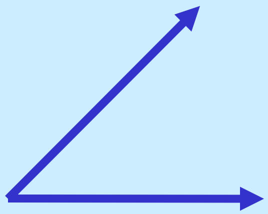
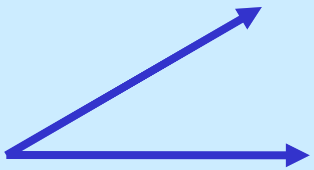
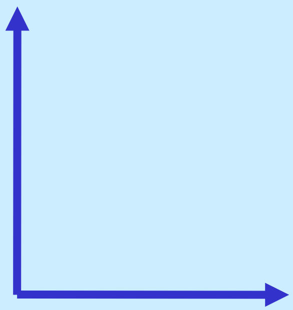
Angles and Their Measure

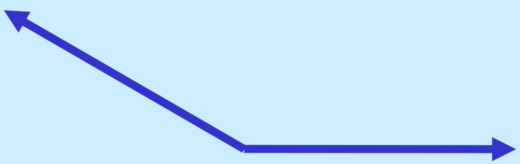
A Sense of Angle Sizes

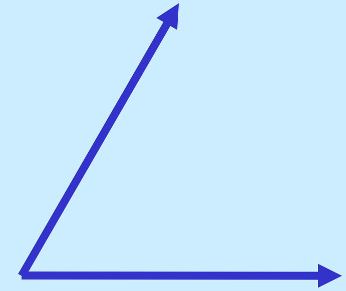
Do Now: See if you can match the angle pictured with the measure in degrees.

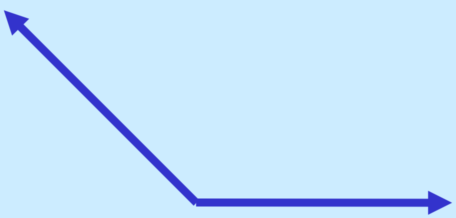






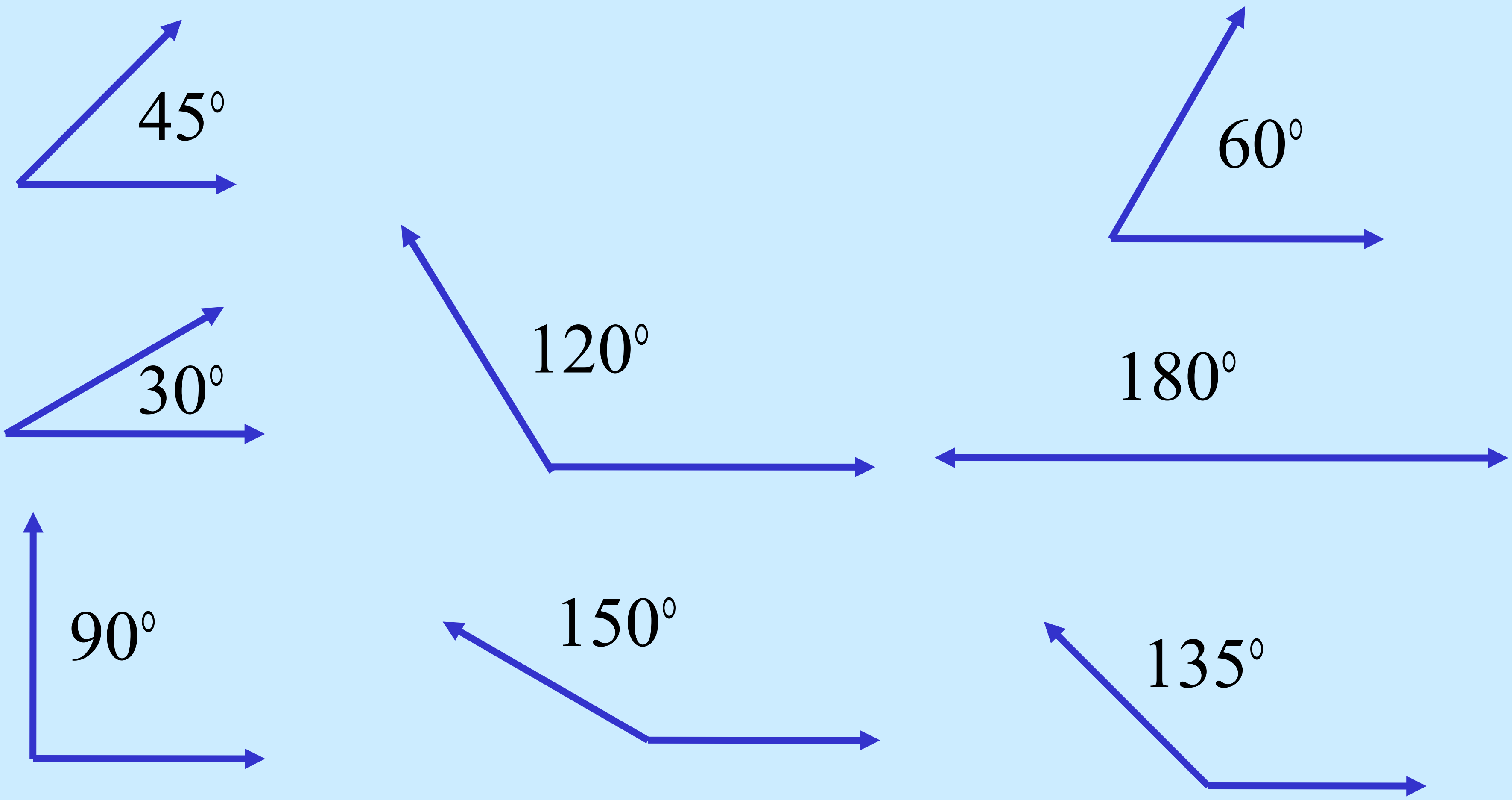






45^{0} 30^{0} 150^{0} 120^{0} 60^{0} 180^{0} 135^{0} 90^{0}

A Sense of Angle Sizes



It's Greek To Me!

It is customary to use small letters in the Greek alphabet to symbolize angle measurement.

α

alpha

β

beta

γ

gamma

θ

theta

ϕ

phi

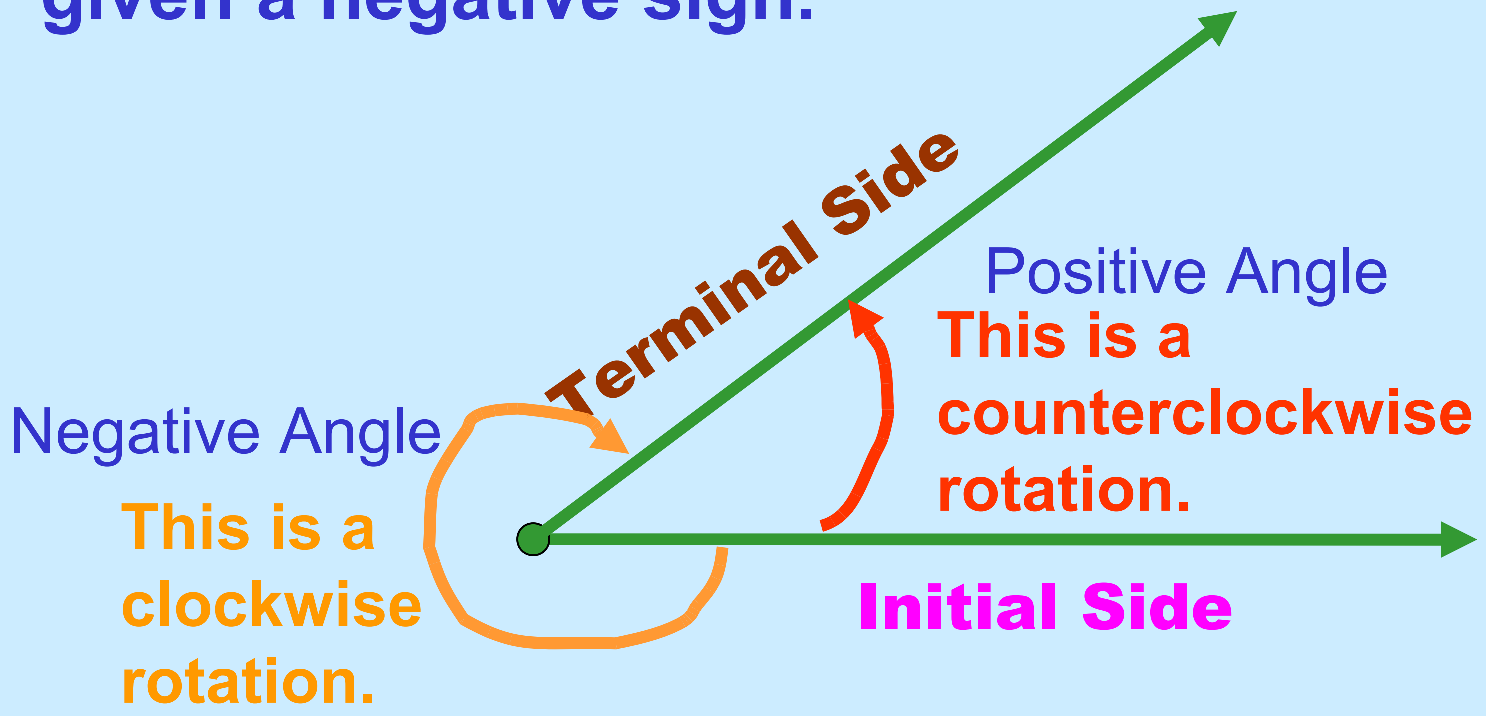
δ

delta

An angle is formed by joining the endpoints of two half-lines called rays.

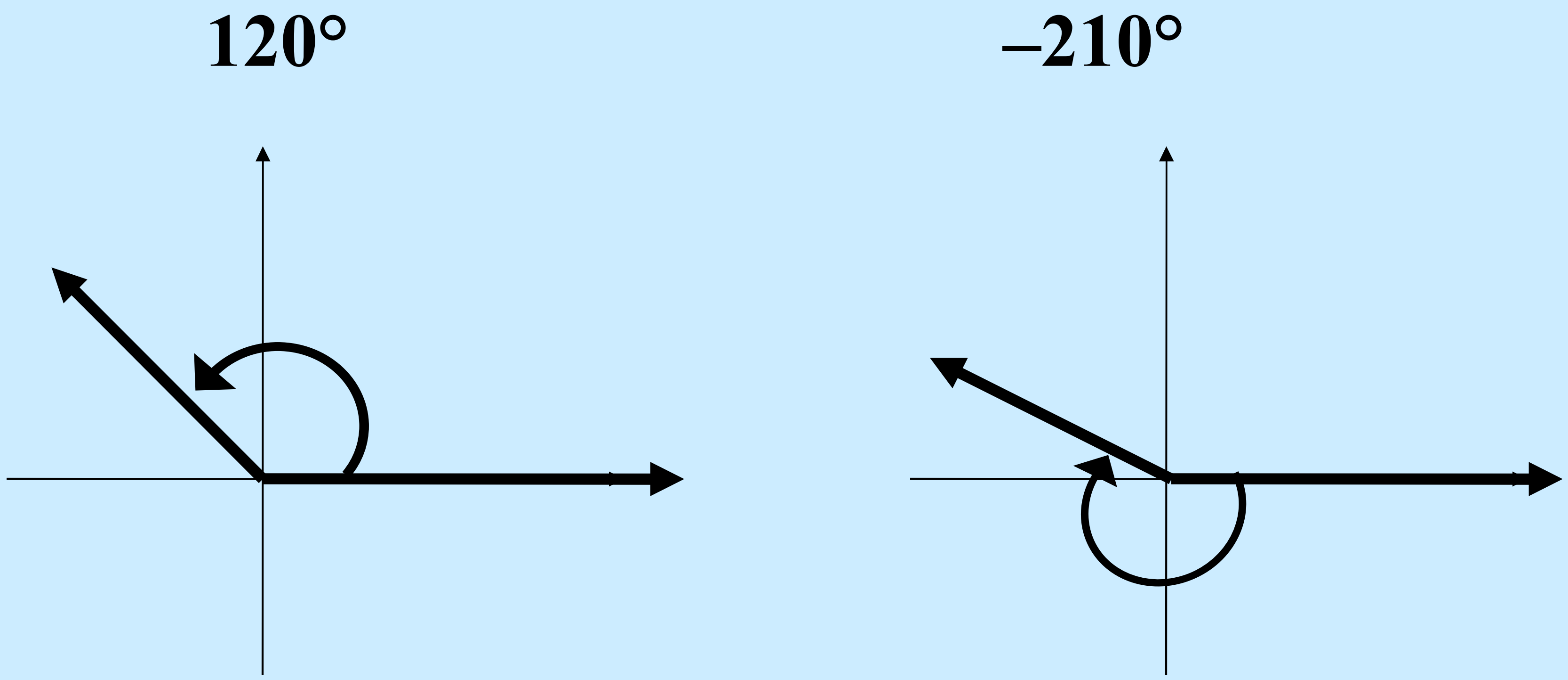
The side you measure to is called the terminal side.

Angles measured counterclockwise are given a positive sign and angles measured clockwise are given a negative sign.



The side you measure from is called the initial side.

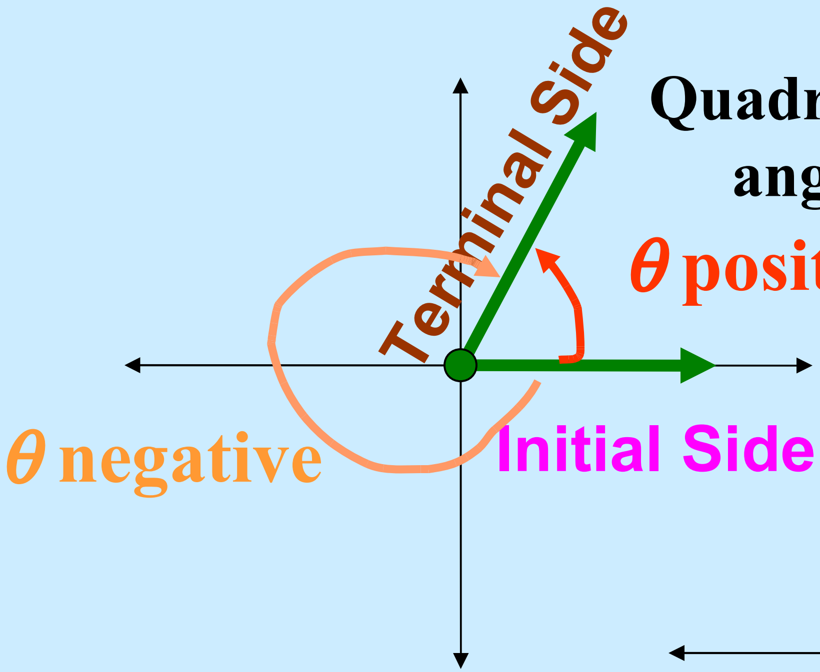
Angle describes the ***amount*** *and* ***direction*** *of rotation*

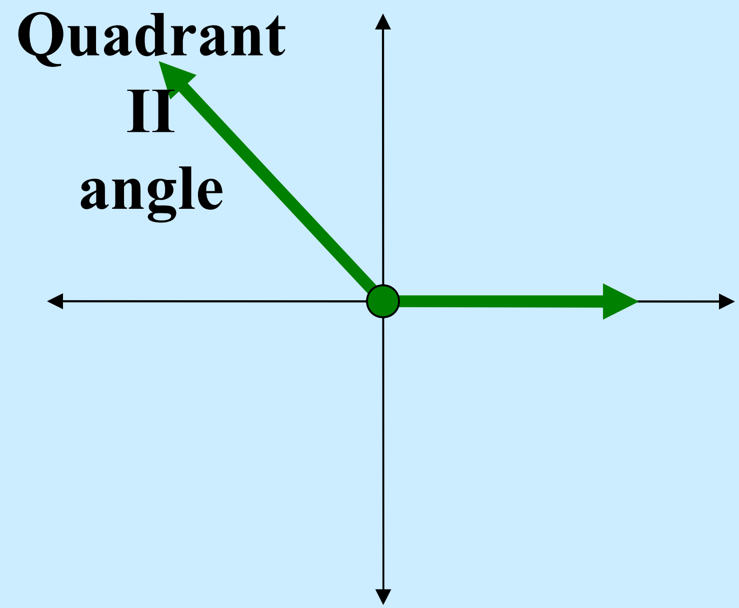


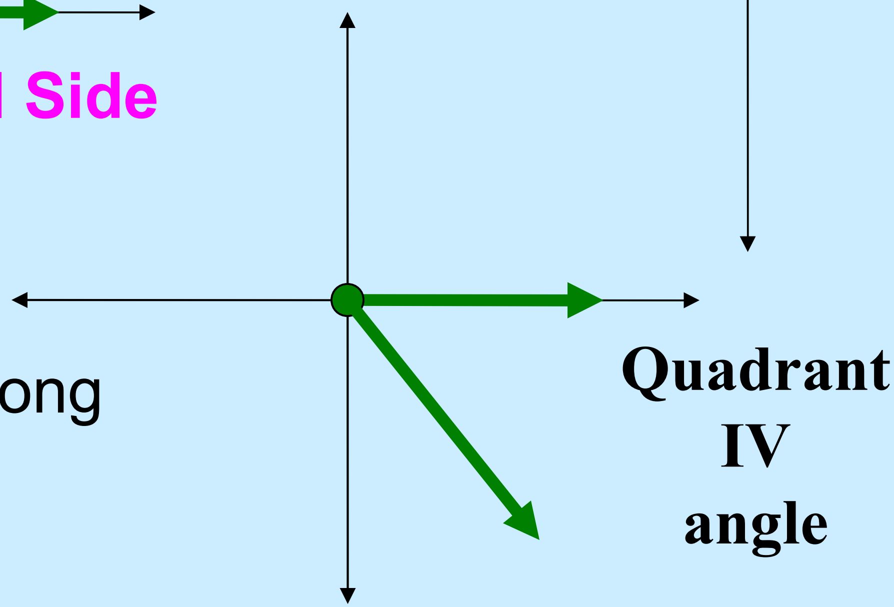
Positive Angle- rotates counter-clockwise (CCW)

Negative Angle- rotates clockwise (CW)

We can use a coordinate system with angles by putting the initial side along the positive *x*-axis with the vertex at the origin.







If the terminal side is along an axis it is called a quadrantal angle.

We say the angle lies in whatever quadrant the terminal side lies in.

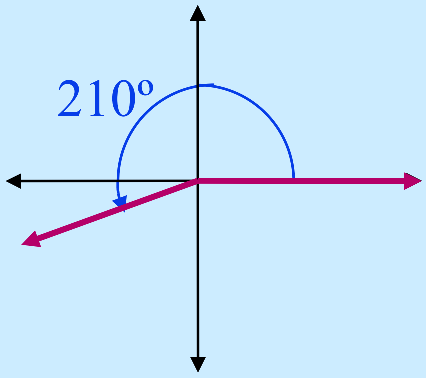
Coterminal Angles: Two angles with the same initial and terminal sides

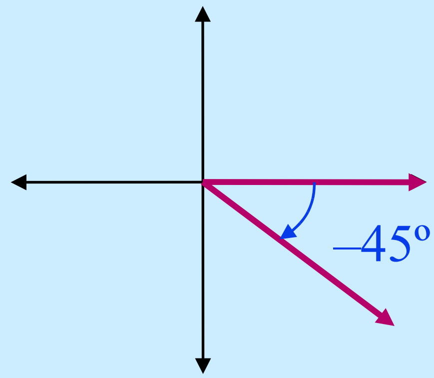
Find a positive coterminal angle to 20º $20 + 360 = 380^0$

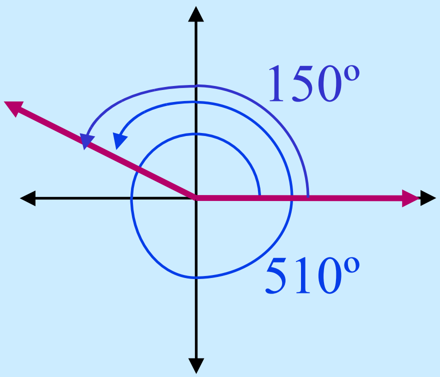
Find a negative coterminal angle to 20º $20 - 360 = -340^0$

Example: Draw an angle with the given measure in standard position. Then determine in which quadrant the terminal side lies.

A. 210^o b. -45^o c. 510^o







Terminal side is in Quadrant III

Terminal side is in Quadrant IV

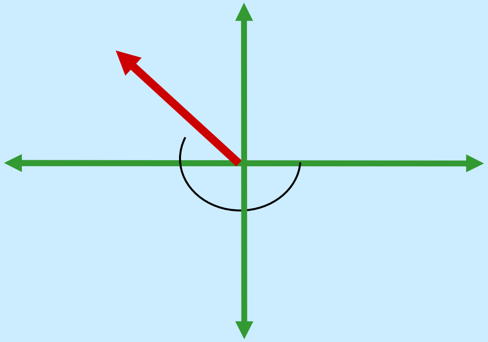
Terminal side is in Quadrant II

Use the fact that $510^o = 360^o + 150^o$. So the terminal side makes 1 complete revolution and continues another 150^o.

510^o and 150^o are called coterminal (their terminal sides coincide). An angle coterminal with a given angle can be found by adding or subtracting multiples of 360^o.

Sketching Angles in Standard Position

d) -220^0



Principal Angle 140^0

Coterminal Angles 500^0

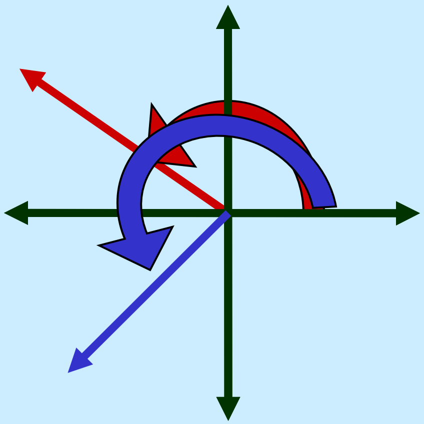
-580^0

To find all coterminal angles:

$\theta = 140 + 360n$ where n is an element of the integers.

Angles in Standard Position

Principal Angle

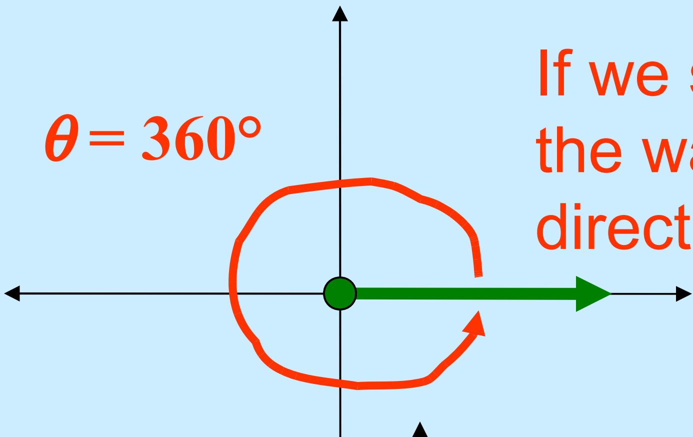


is measured from the positive *x*-axis to the terminal arm.

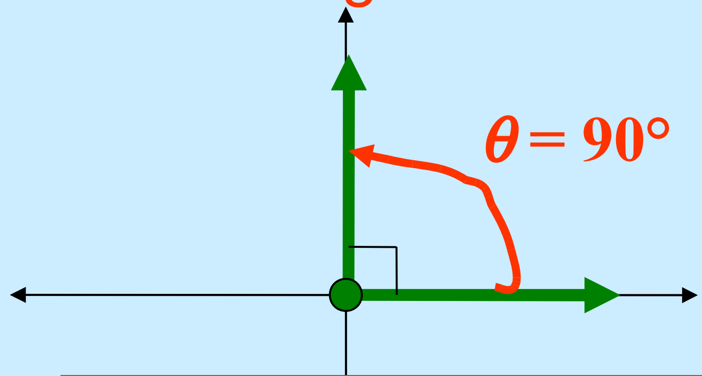
is measured in a counterclockwise direction, therefore is always positive.

is always less than 360^0.

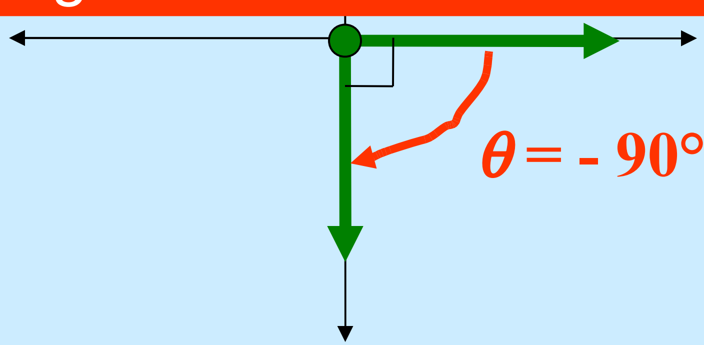
We will be using two different units of measure when talking about angles: Degrees and Radians



If we start with the initial side and go all of the way around in a counterclockwise direction we have 360 degrees



If we went 1/4 of the way in a clockwise direction the angle would measure -90°



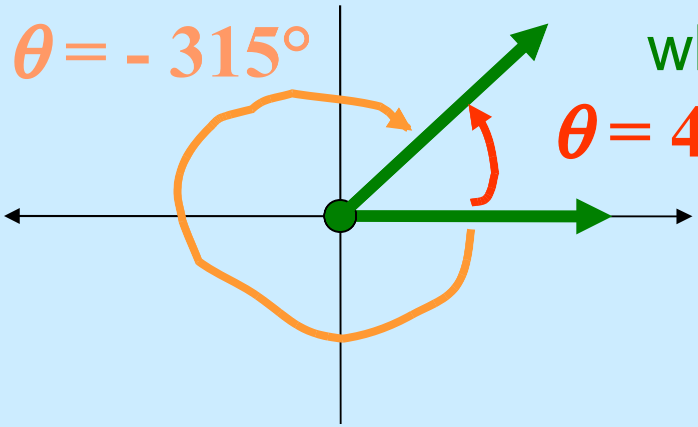
You are probably already familiar with a right angle that measures 1/4 of the way around or 90°

Let's talk about degrees first. You are probably already somewhat familiar with degrees.

What is the measure of this angle?

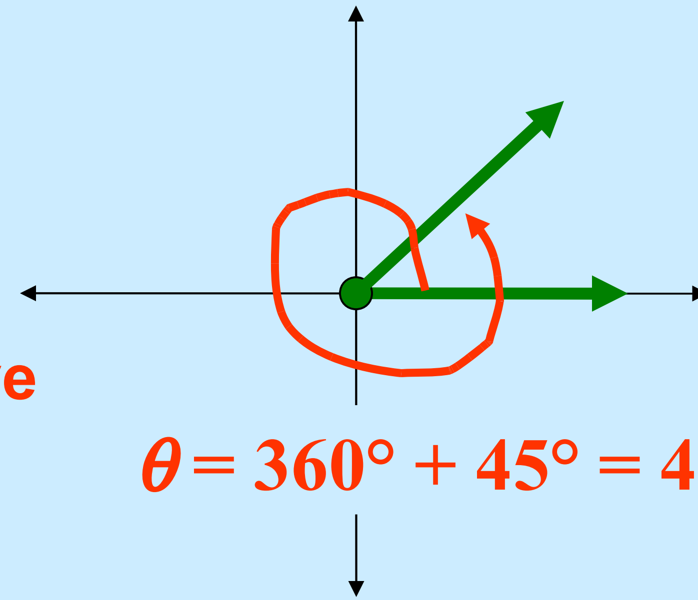
$\theta = -360° + 45°$

$\theta = -315°$



$\theta = 45°$

You could measure in the positive direction and go around another rotation which would be another 360°



You could measure in the positive direction

You could measure in the negative direction

There are many ways to express the given angle. Whichever way you express it, it is still a Quadrant I angle since the terminal side is in Quadrant I.

If the angle is not exactly to the next degree it can be expressed as a decimal (most common in math) or in degrees, minutes and seconds (common in surveying and some navigation).

1 degree = 60 minutes   1 minute = 60 seconds

$$\theta = 25°48'30''$$

degrees minutes seconds

To convert to decimal form use conversion fractions. These are fractions where the numerator = denominator but two different units. Put unit on top you want to convert to and put unit on bottom you want to get rid of.

Let's convert the seconds to minutes

$$30'' \cdot \frac{1'}{60''} = 0.5'$$

1 degree = 60 minutes 1 minute = 60 seconds

$$\theta = 25°48'30'' = 25°48.5' = 25.808°$$

Now let's use another conversion fraction to get rid of minutes.

$$48.5' \cdot \frac{1°}{60'} = .808°$$