Factoring Special Quadratics

Do Now: Find the product:

1. $(x+5)(x+5)$
2. $(x+6)(x-6)$
3. $2(x+5)(x-5)$

Difference of Squares

Recall that, when multiplying conjugate binomials, the product is a difference of squares.

E.g., $(x - 7)(x + 7) = x^2 - 49$

> Therefore, when factoring a difference of squares, the factors will be conjugate binomials.

Factor:

$x^2 - 81 = (x - 9)(x + 9)$

$(x)^2 - (9)^2$

$16x^2 - 121 = (4x - 11)(4x + 11)$

$(4x)^2 - (11)^2$

$5x^2 - 80 = 5(x^2 - 16)$

$= 5(x - 4)(x + 4)$

Factoring Completely

$x^2 + 25$ CANNOT BE FACTORED

Factor completely:

$x^4 - 16 = (x^2 - 4)(x^2 + 4)$
$= (x - 2)(x + 2)(x^2 + 4)$

$x^8 - 1 = (x^4 - 1)(x^4 + 1)$
$= (x^2 - 1)(x^2 + 1)(x^4 + 1)$
$= (x - 1)(x + 1)(x^2 + 1)(x^4 + 1)$

$x^4 - 13x^2 + 36 \quad = (x^2 - 9)(x^2 - 4)$
$= (x - 3)(x + 3) \quad (x - 2)(x + 2)$

Factoring a Difference of Squares with a Complex Base

Factor completely:

$$(x+y)^2 - 16 = [(x+y) - 4][(x+y) + 4]$$
$$= (x + y - 4)(x + y + 4)$$

$$(x+5)^2 - 49 = [(x+5) - 7][(x+5) + 7]$$
$$= (x + 5 - 7)(x + 5 + 7)$$
$$= (x - 2)(x + 12)$$

$$(3x+2)^2 - 81 = [(3x+2) - 9][(3x+2) + 9]$$
$$= (3x + 2 - 9)(3x + 2 + 9)$$
$$= (3x - 7)(3x + 11)$$

$$25(x+4)^2 - 49 = [5(x+4) - 7][5(x+4) + 7]$$
$$= (5x + 20 - 7)(5x + 20 + 7)$$
$$= (5x + 13)(5x + 27)$$

Factoring a Difference of Squares with a Complex Base

$$16 - (x - y)^2 = [4 - (x - y)]\,[4 + (x - y)]$$
$$= (4 - x + y)(4 + x - y)$$

$$25 - (x + 3)^2 = [5 - (x + 3)]\,[5 + (x + 3)]$$
$$= (5 - x - 3)(5 + x + 3)$$
$$= (-x + 2)(8 + x)$$

$$81 - (3x + 2)^2 = [9 - (3x + 2)][9 + (3x + 2)]$$
$$= (9 - 3x - 2)(9 + 3x + 2)$$
$$= (-3x + 7)(3x + 11)$$

$$49 - 25(x + 4)^2 = [7 - 5(x + 4)]\,[7 + 5(x + 4)]$$
$$= (7 - 5x - 20)(7 + 5x + 20)$$
$$= (-5x - 13)(5x + 27)$$

Perfect Square Trinomials

Recall: $(a + 3)^2 = a^2 + 6a + 9$

The middle term is twice the product of the two terms of the binomial: $2 \times 3 \times a = 6a$

The first and last terms are perfect squares.

Factor:

$a^2 + 8a + 16 = (a + 4)^2$

Same sign

The square root of a^2 is a.

The square root of 16 is 4.

Check:
The middle term is twice the product of the two terms:
$2(a)(4) = 8a$

Perfect Square Trinomials

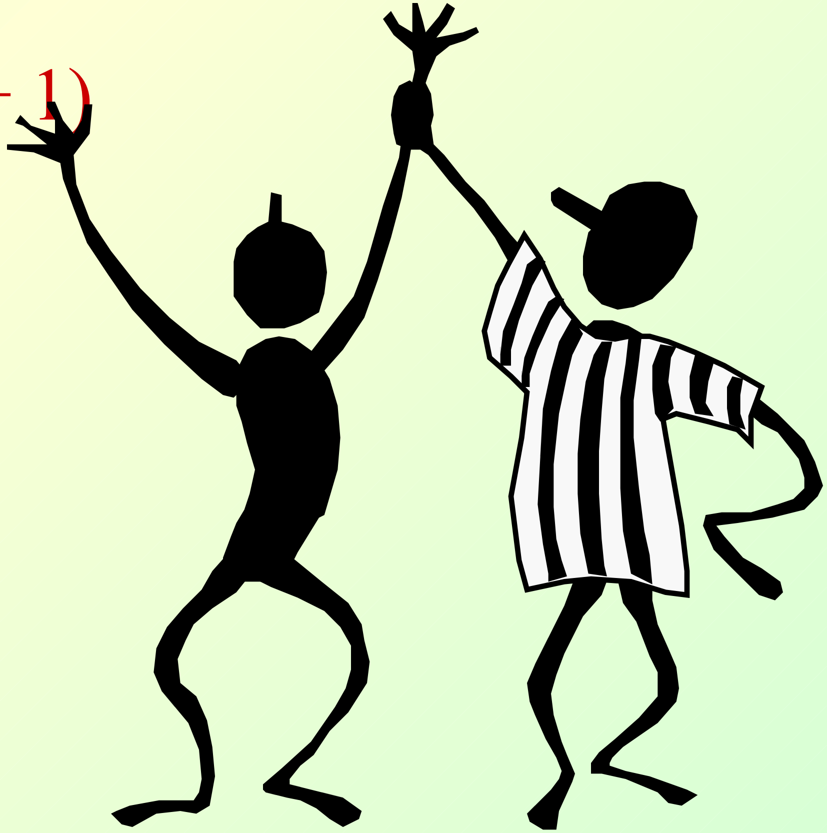
$9a^2 + 120a + 400 = (3a + 20)^2$

$81a^2 - 72a + 16 = (9a - 4)^2$

$100a^2 - 140a + 49 = (10a - 7)^2$

$28a^2 + 28a + 7 = 7(4a^2 + 4a + 1)$

$= 7(2a + 1)^2$



How do you factor the sum or difference of cubes?

You'll need to memorize the factorization of the sum or difference of two cubes:

Sum of Two Cubes:

$$\mathbf{a^3 + b^3} = (a + b)(a^2 - ab + b^2)$$

Difference of Two Cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Example: *Factor* $x^3 + 27$

The cube roots of the terms are x *and* 3

$$= (x + 3)(x^2 - 3x + 9)$$

Example: Factor $128x^3 - 250$

Factor out the GCF first…

$$= 2(64x^3 - 125)$$

The cube roots are $4x$ *and* -5

$$= 2(4x - 5)(16x^2 + 20x + 25)$$