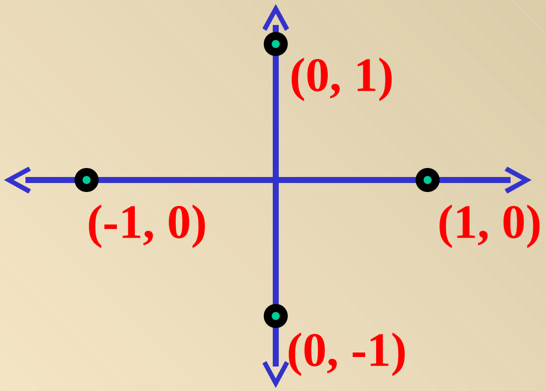
Graphing Sine and Cosine Functions

Periodic Functions

Functions that repeat themselves over a particular interval of their domain are periodic functions. The interval is called the period of the function.

The amplitude of a periodic function is one half the distance between the maximum and minimum values.

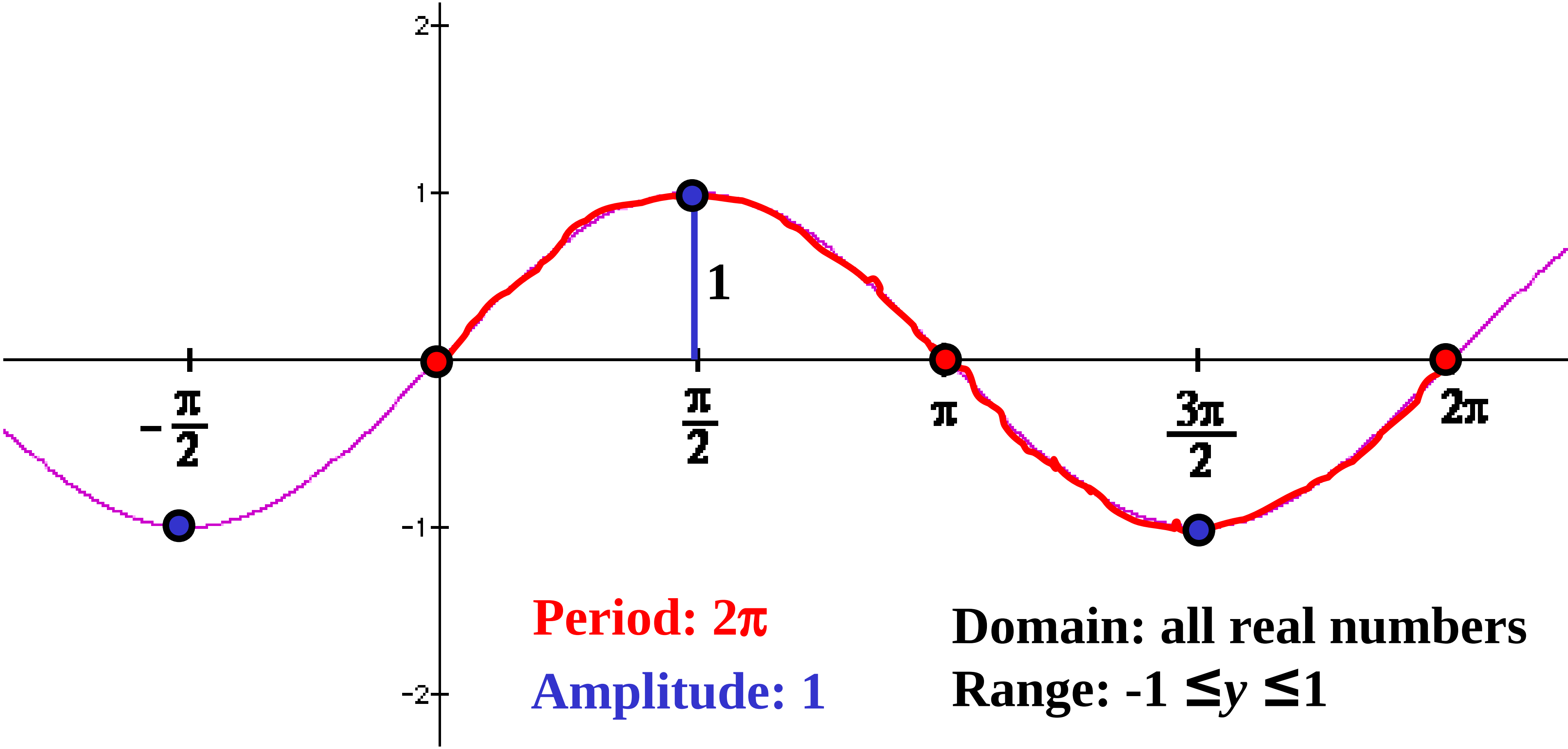
To graph a periodic function such as sin *x*, use the exact values of the angles of 30^0, 45^0, and 60^0. In particular, keep in mind the quadrantal angles of the unit circle.



The points on the unit circle are in the form (cosine, sine).

Graphing a Periodic Function

Graph $y = \sin x$.



Period: 2π

Amplitude: 1

Domain: all real numbers

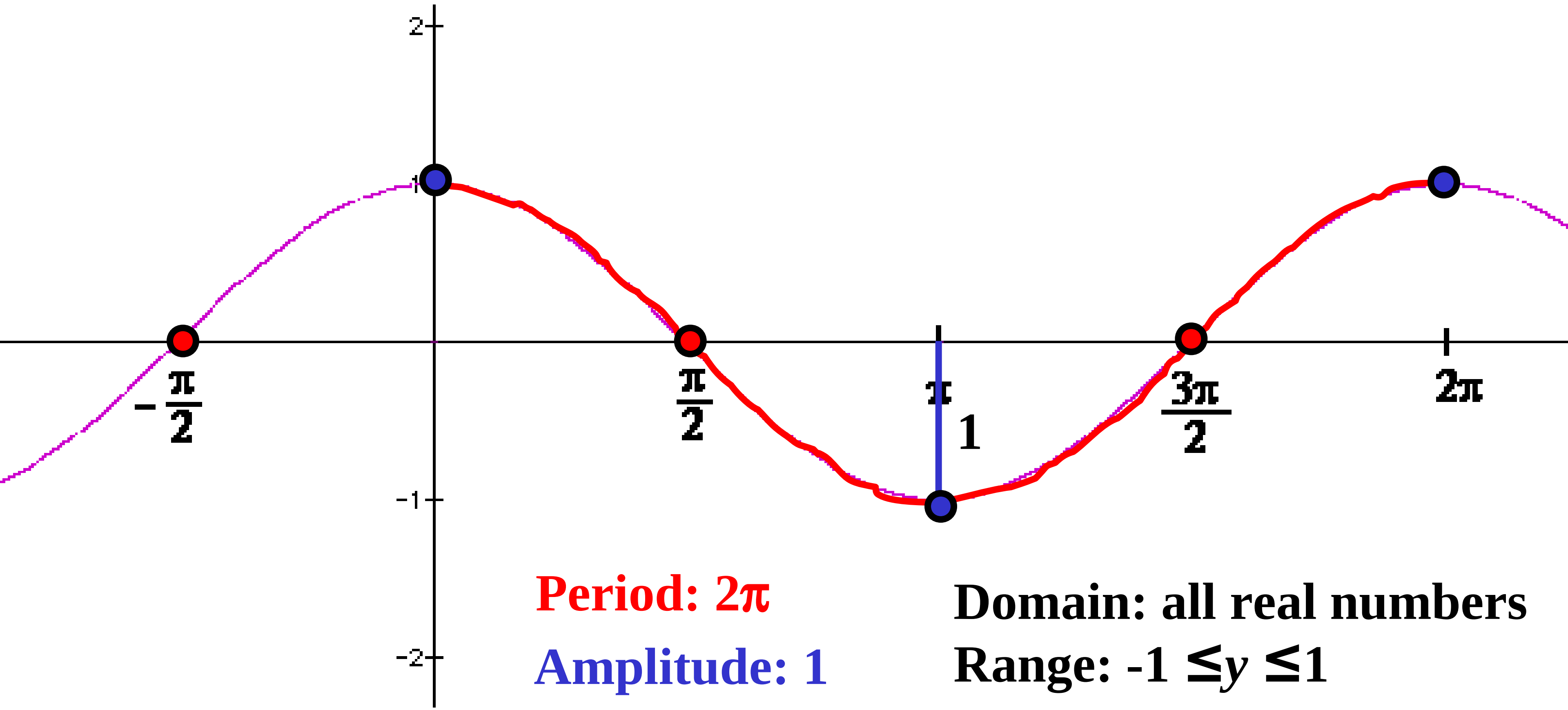
Range: $-1 \leq y \leq 1$

***y*-intercept: 0**

***x*-intercepts: 0, ±π, ±2π, ...**

Graphing a Periodic Function

Graph $y = \cos x$.



Period: 2π

Amplitude: 1

Domain: all real numbers

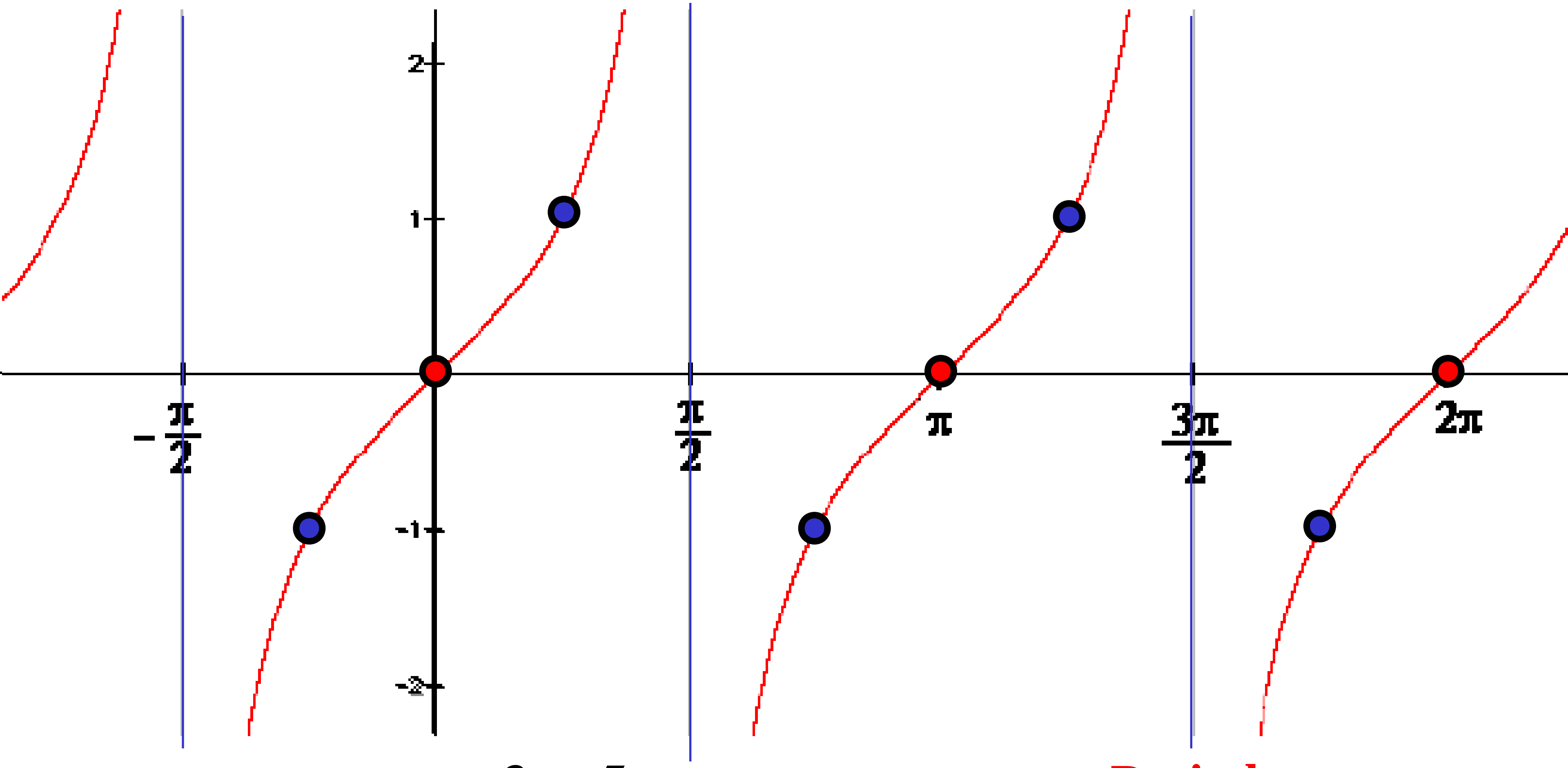
Range: $-1 \leq y \leq 1$

y-intercept: 1

x-intercepts: $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$, ...

Graphing a Periodic Function

Graph $y = \tan x$.



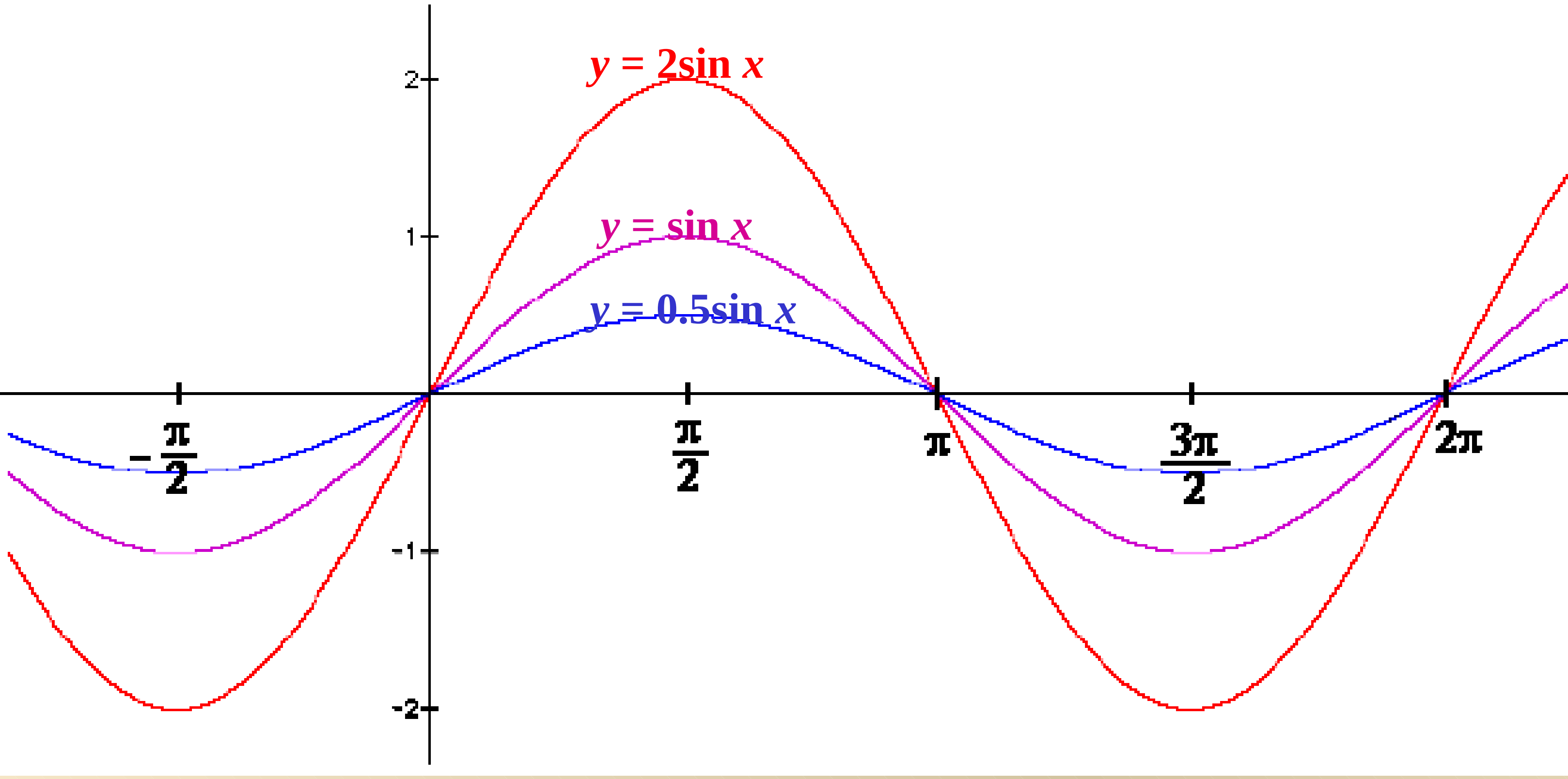
Asymptotes: $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots, \frac{\pi}{2} + \pi n,\ n \in I$

Period: π

Domain: $\{x \mid x \neq \frac{\pi}{2} + \pi n,\ n \in I,\ x \in R\}$

Range: all real numbers

Determining the Amplitude of $y = a \sin x$



Comparing the Graphs of $y = a \sin x$

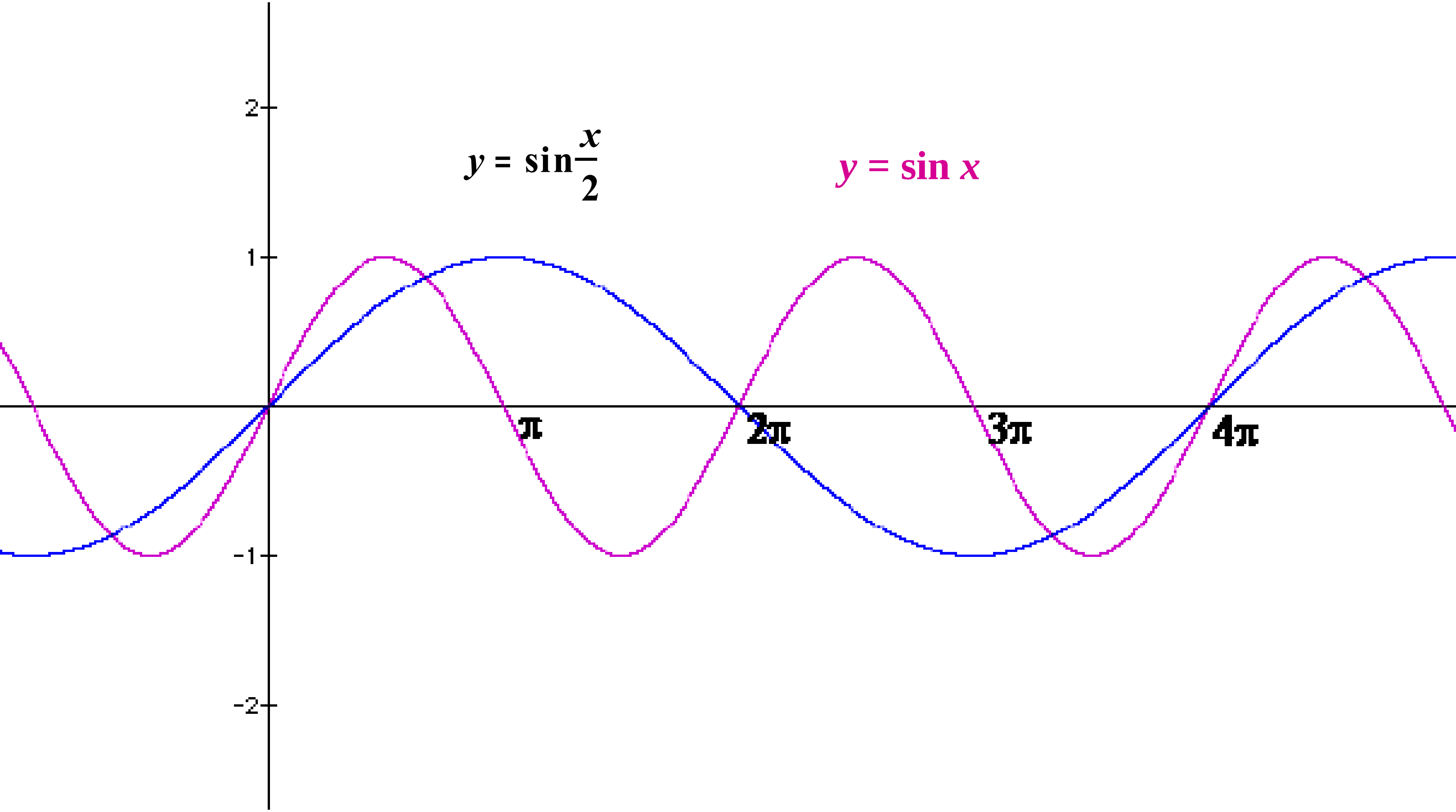
	$y = \sin x$	$y = 2\sin x$	$y = 0.5\sin x$
Period	2π	2π	2π
Amplitude	1	2	0.5
Domain	all real numbers	all real numbers	all real numbers
Range	$-1 \leq y \leq 1$	$-2 \leq y \leq 2$	$-0.5 \leq y \leq 0.5$

The amplitude of the graph of $y = a \sin x$ is $|a|$.

When $a > 1$, there is a vertical stretch by a factor of a.

When $0 < a < 1$, there is a vertical compression by a factor of a.

Determining the Period for $y = \sin bx$, $b > 0$



Comparing the Graphs of $y = \sin bx$

	$y = \sin x$	$y = \sin 2x$	$y = \sin 0.5x$
Period	2π	π	4π
Amplitude	1	1	1
Domain	all real numbers	all real numbers	all real numbers
Range	$-1 \leq y \leq 1$	$-1 \leq y \leq 1$	$-1 \leq y \leq 1$

The period for $y = \sin bx$ is $\dfrac{2\pi}{b}$, $b > 0$.

When $b > 1$, there is a horizontal compression.
When $0 < b < 1$, there is a horizontal expansion.

Determining the Period and Amplitude of $y = a \sin bx$

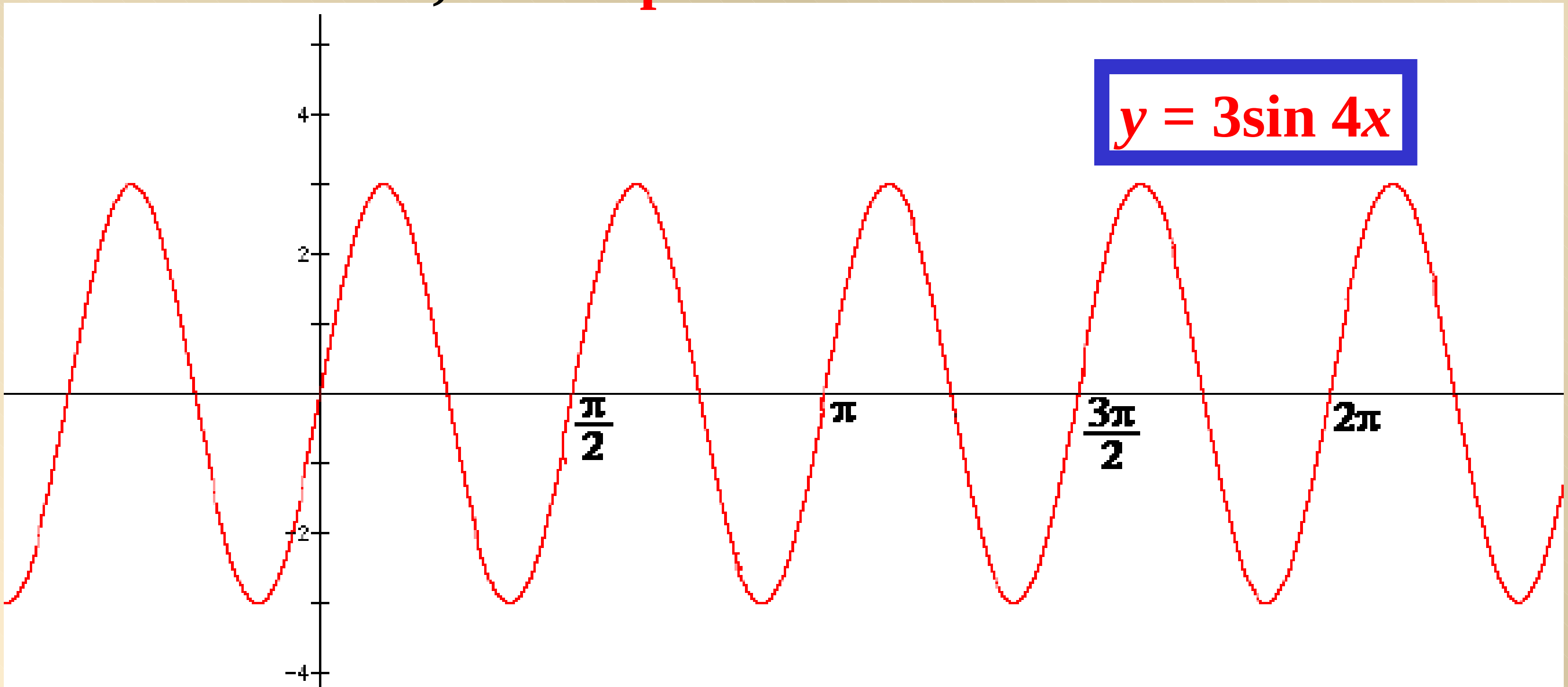
Given the function $y = 3\sin 4x$, determine the period and the amplitude.

The period of the function is $\dfrac{2\pi}{b}$.

Therefore, the period is $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$.

The amplitude of the function is $|a|$.

Therefore, the amplitude is 3.

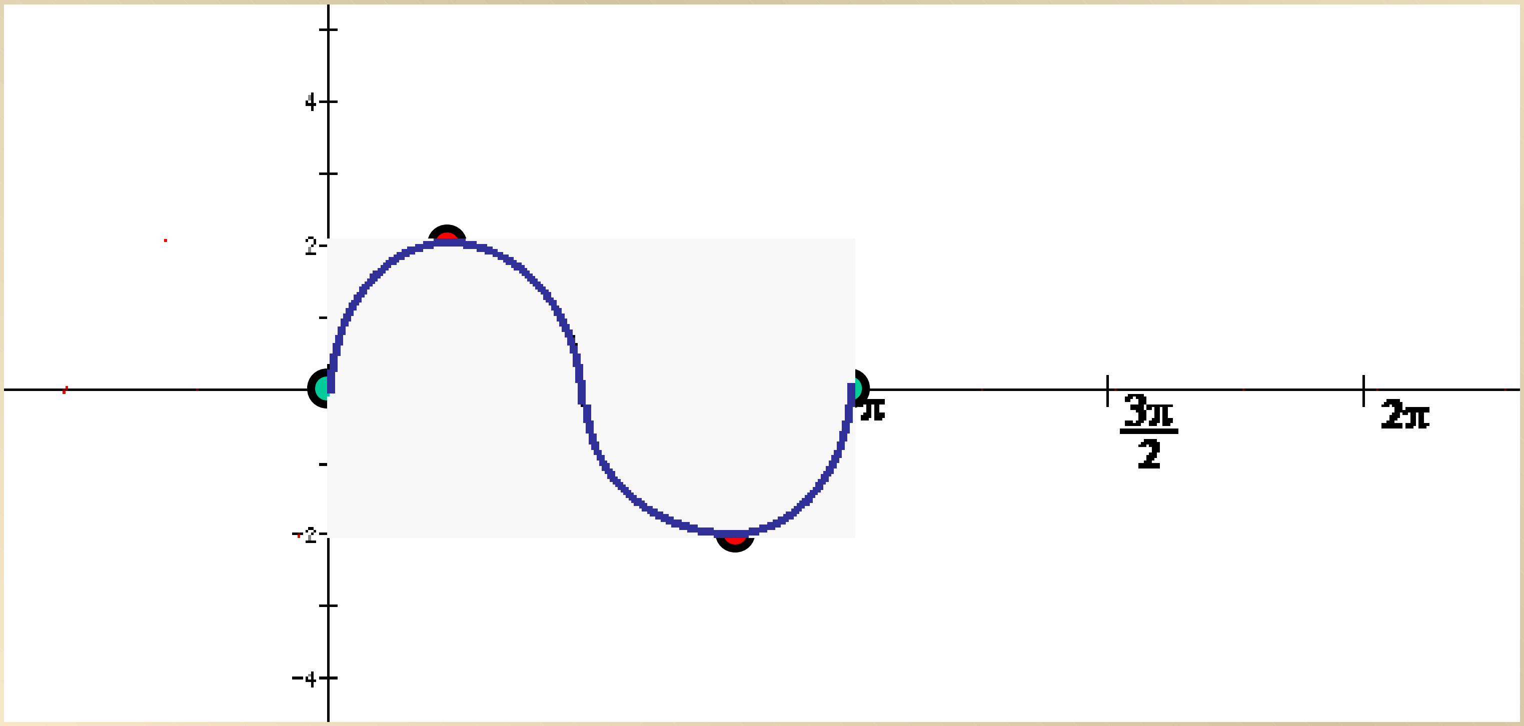


Determining the Period and Amplitude of $y = a \sin bx$

Sketch the graph of $y = 2\sin 2x$.

The period is π.

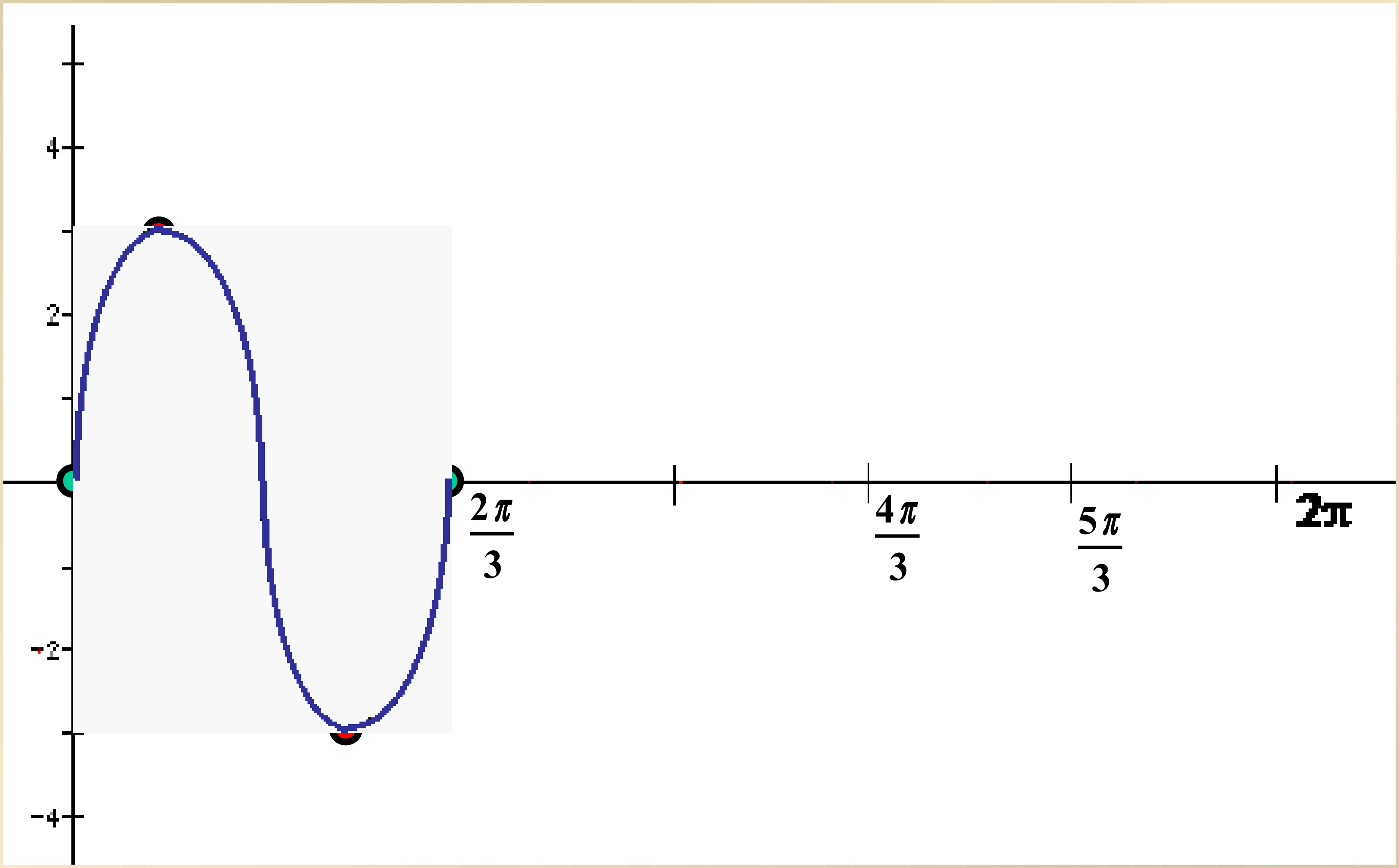
The amplitude is 2.



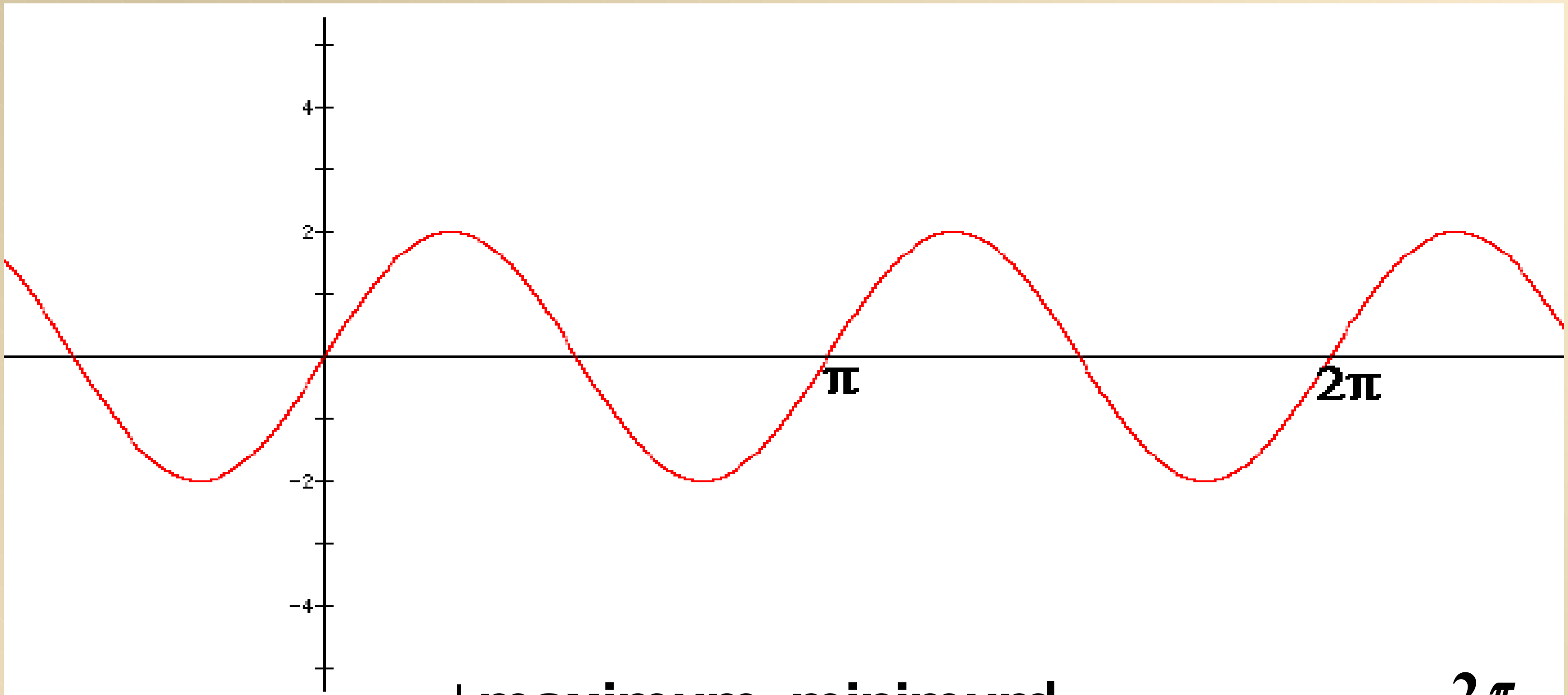
Determining the Period and Amplitude of $y = a \sin bx$

Sketch the graph of $y = 3\sin 3x$.

The period is $\frac{2\pi}{3}$. The amplitude is 3.



Writing the Equation of the Periodic Function



Amplitude $= \dfrac{|\text{maximum} - \text{minimum}|}{2}$

$= \dfrac{|2 - (-2)|}{2}$

$= 2$

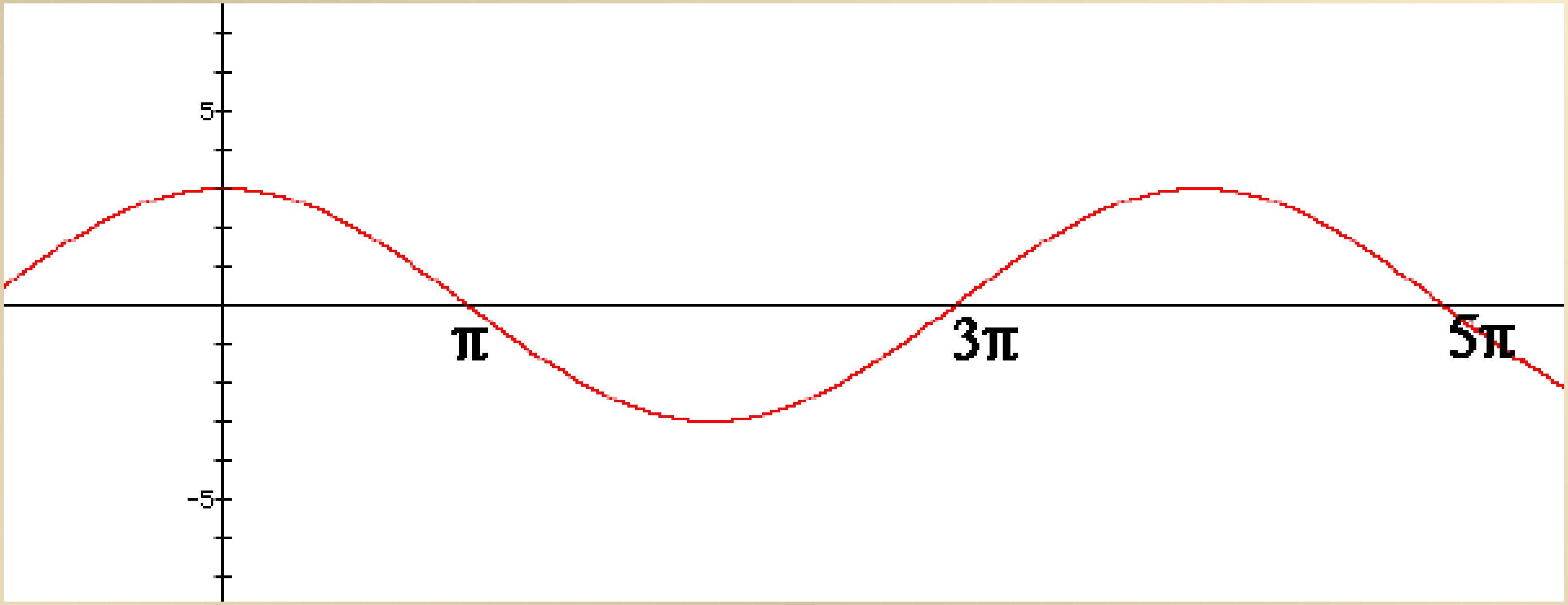
Period $= \dfrac{2\pi}{b}$

$\pi = \dfrac{2\pi}{b}$

$b = 2$

Therefore, the equation as a function of sine is
$y = 2\sin 2x$.

Writing the Equation of the Periodic Function



$$\textbf{Amplitude} = \frac{|\text{maximum} - \text{minimum}|}{2}$$

$$= \frac{|3-(-3)|}{2}$$

$$= 3$$

$$\text{Period} = \frac{2\pi}{b}$$

$$4\pi = \frac{2\pi}{b}$$

$$b = 0.5$$

Therefore, the equation as a function of cosine is $y = 3\cos 0.5x$.