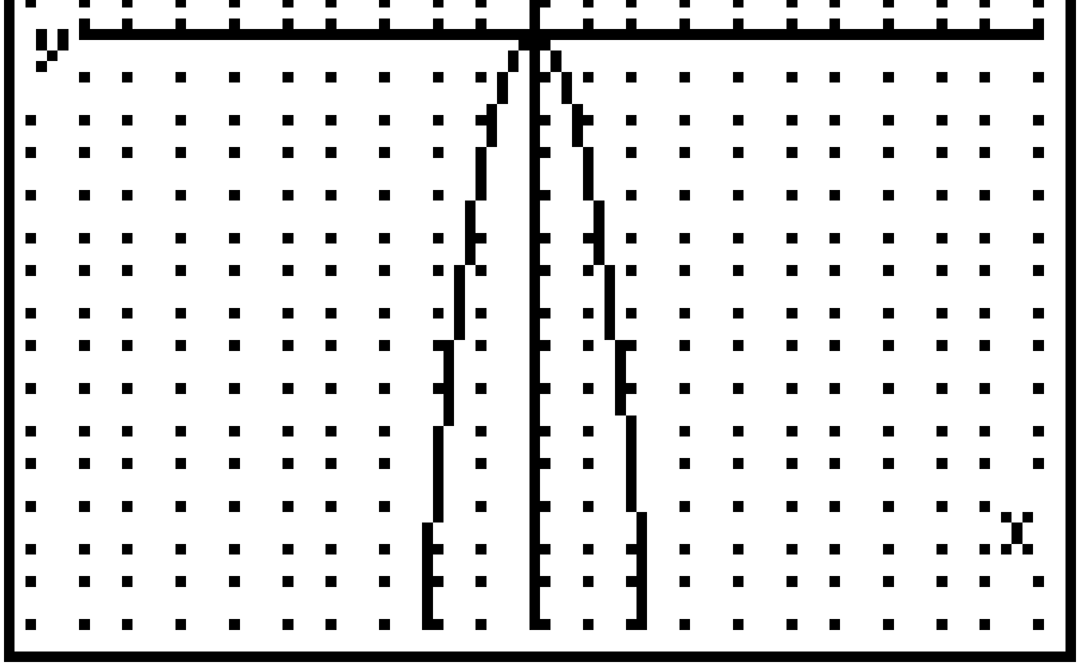
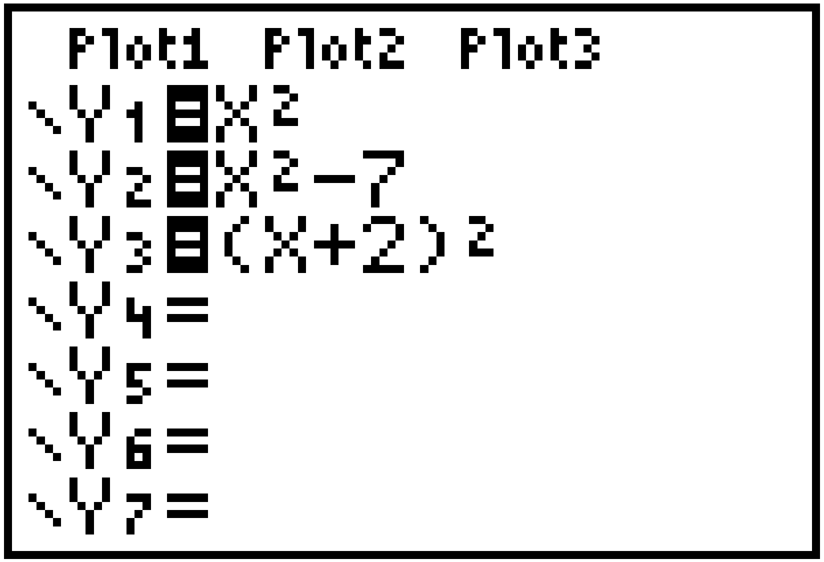
How To Graph Quadratic Equations

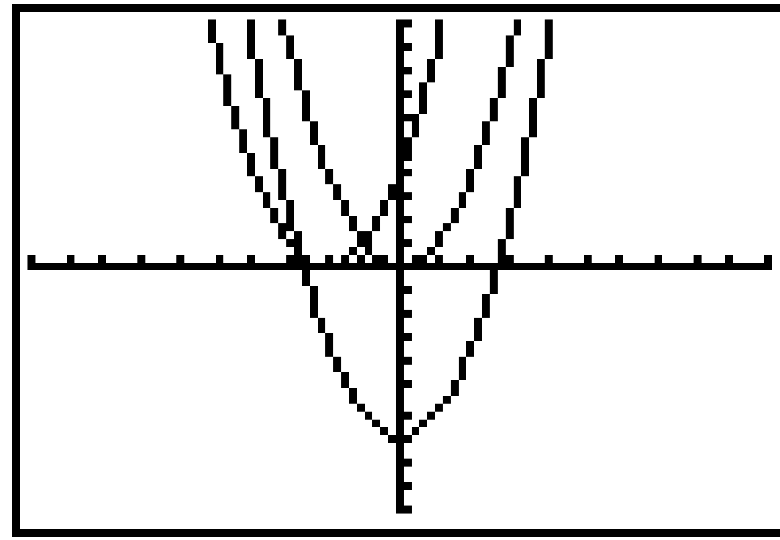


Do Now:

Using your graphing calculator, compare the graphs of the following quadratic equations to each other.

x^2, $x^2 - 7$, $(x + 2)^2$





- ❖ All three graphs have the same shape.
- ❖ The vertex of the graph of $x^2 - 7$ will move down 7 on the y-axis.
- ❖ the vertex of the graph of $(x+2)^2$ will move left two on the x-axis.

Given the following function,

$$y = x^2 + a$$

If: a > 0, then shift the graph “a” units up

If:a< 0, then shift the graph “a” units down

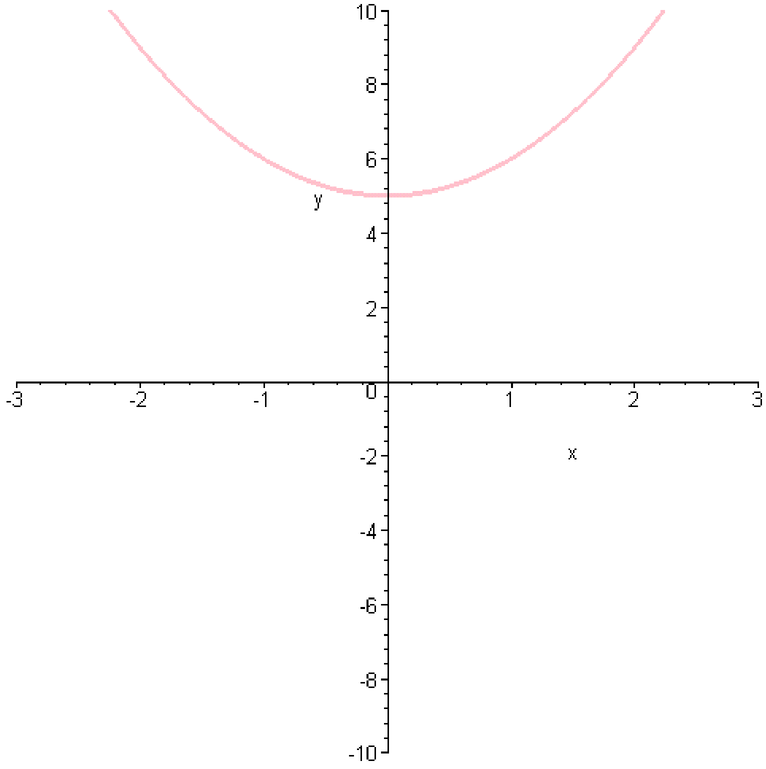
Given the following function,

$$y = x^2 + 5$$

Since a > 0, then the graph will be shift up “5” units

Let's graph

$$y = x^2 + 5$$



Given the following function,

$$y = x^2 - 3$$

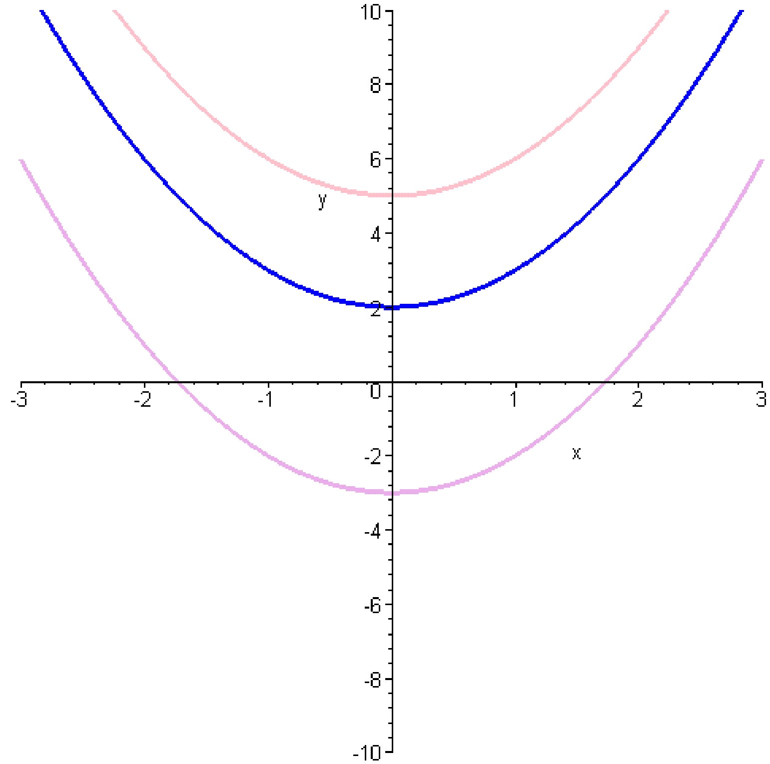
Since a < 0, then the graph will be shift down "3" units

Let's graph

$y = x^2 + 5$

$y = x^2 + 2$

$y = x^2 - 3$



Given the following function,

$$y = (x - b)^2$$

For this equation, b is inside the parenthesis.

We get the expression and equal it to zero.

$$x - b = 0$$

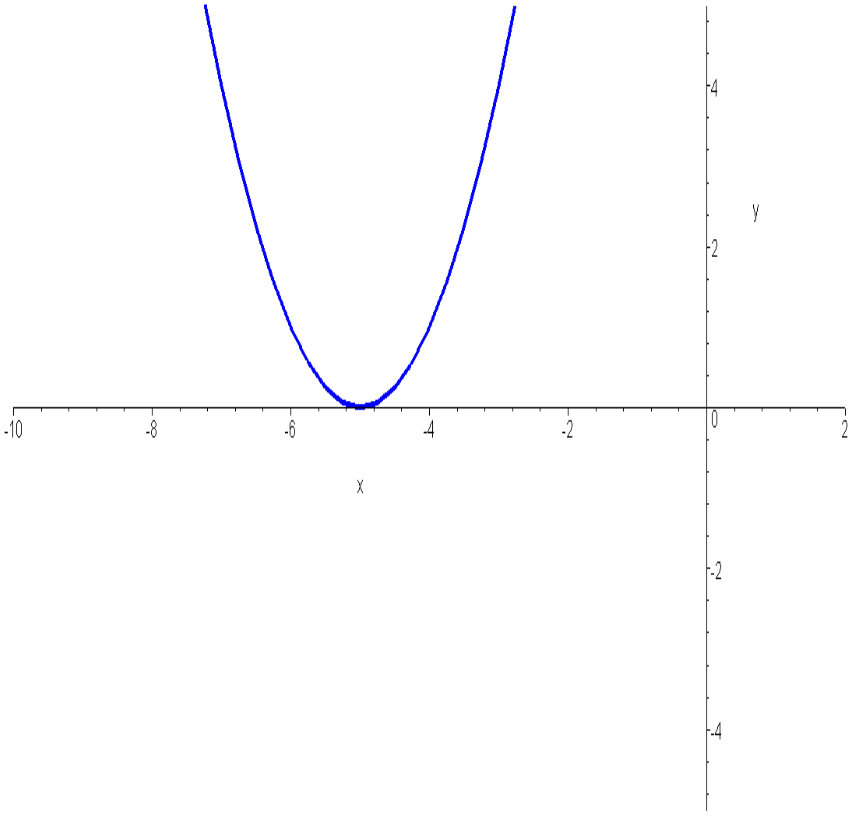
$$x = b$$

If: $b > 0$, then shift the graph “b” units right

If: $b < 0$, then shift the graph “b” units left

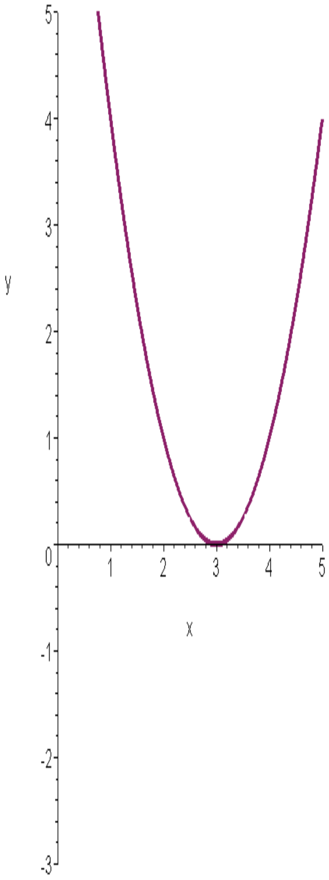
Let's graph

$$y = (x + 5)^2$$



Let's graph

$$y = (x - 3)^2$$



Recall: $y = (x - b)^2 + a$

a > 0 then shift up

a < 0 then shift down

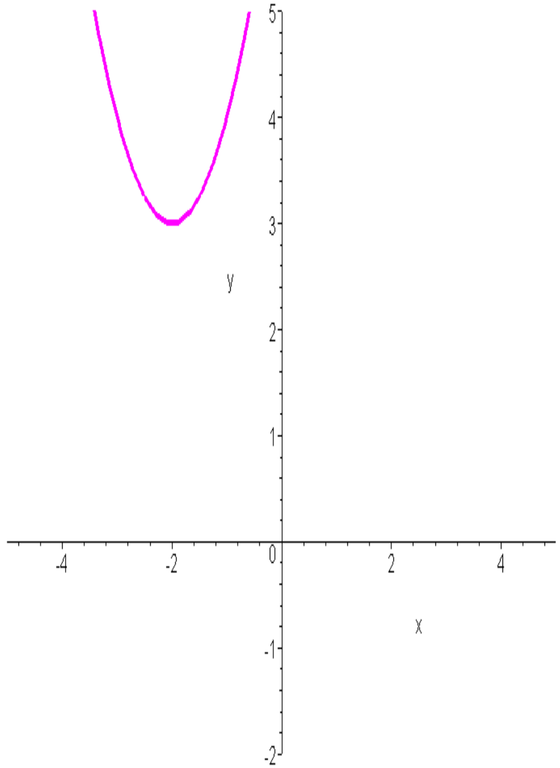
Equal the expression to zero

$$y = (x + 2)^2 + 3$$

$$(x + 2) = 0$$

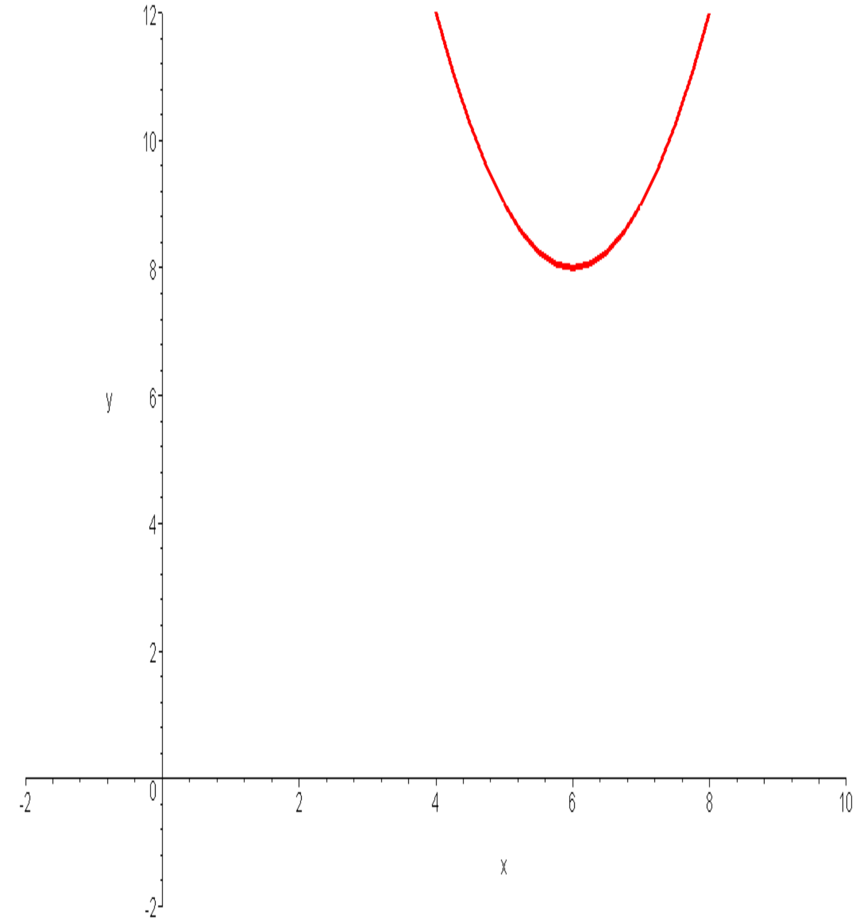
b > 0 then shift to the right

b < 0 then shift to the left



Let's graph

$$y = (x - 6)^2 + 8$$



Given the following function,

$$y = cx^2$$

For this equation, c determines how wide or thin the parabola will be.

if: $|c|>1$, then the graph is closer to the y-axis

if: $|c|=1$, then the graph remains the same

if: $0<|c|<1$, then the graph is further from the y-axis

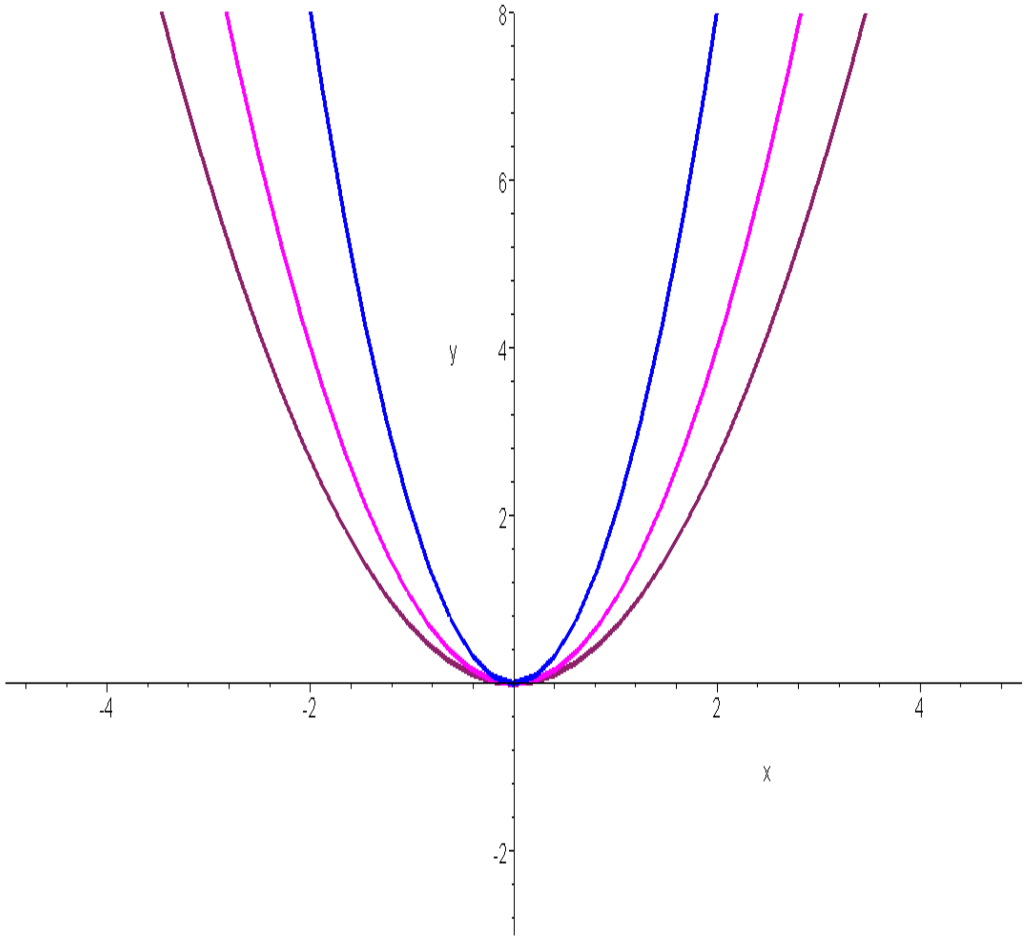
if c is a negative number, then the graph will reflect on the x-axis

Let's graph

$$y = x^2$$

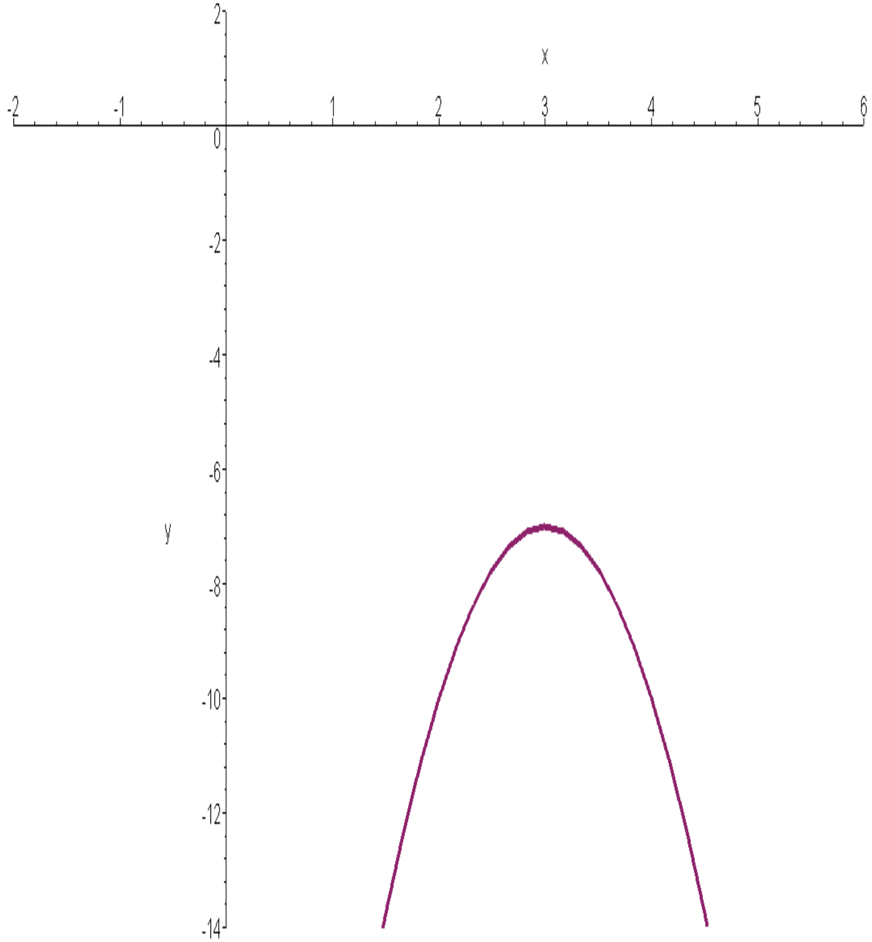
$$y = 2x^2$$

$$y = \frac{2}{3}x^2$$



Let's graph

$$y = -3(x-3)^2 - 7$$



Getting Started

1. The standard form of a **quadratic equation** is $y = ax^2 + bx + c$.
2. The graph of a quadratic equation is a **parabola**.
3. When ***a*** is **positive**, the graph opens **up**.
4. When ***a*** is **negative**, the graph opens **down**.
5. Every parabola has a **vertex**. For graphs opening up, the vertex is a minimum (low point). For graphs opening down, the vertex is a maximum (high point).
6. The ***x-coordinate*** of the vertex is equal to $-\frac{b}{2a}$.

Find the Vertex

To find the *x* coordinate of the vertex, use the equation

$$x = -\frac{b}{2a}$$

Then substitute the value of *x* back into the equation of the parabola and solve for *y*.

You are given the equation $y=-x^2 + 4x - 1$. Find the coordinates of the vertex.

$$a = -1, \quad b = 4$$

$$x = -\frac{b}{2a}$$

$$x = -\frac{4}{(2)(-1)}$$

$$x = 2$$

Substitute and solve for *y*

$$y = -x^2 + 4x - 1$$

$$y = -(2)^2 + 4(2) - 1$$

$$y = -4 + 8 - 1$$

$$y = 3$$

The coordinates of the vertex are (2,3)

Table of Values

- **Choose two values of *x* that are to the right or left of the *x*-coordinate of the vertex.**
- **Substitute those values in the equation and solve for *y*.**
- **Graph the points. (Keep in mind the value of *a* as this will help you determine which way the graph opens.)**
- **Since a parabola is symmetric about the vertical line through the vertex, you can plot mirror image points with the same *y*-values on the "other side" of the parabola.**

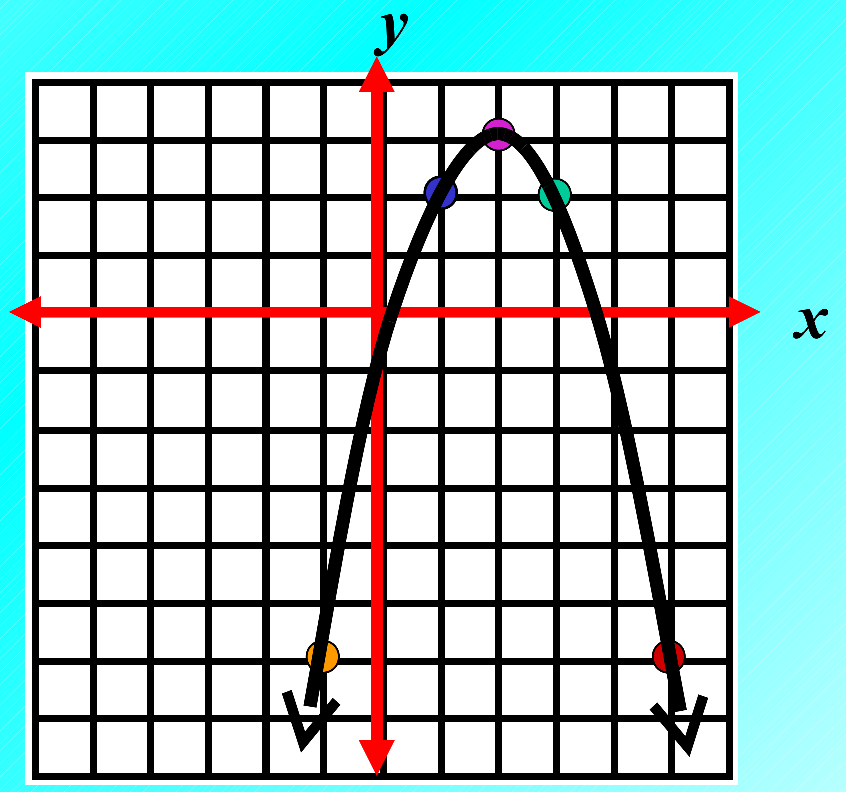
x	$y = -x^2 + 4x - 1$	y
1	$y = -(1)^2 + 4(1) - 1$ $y = -1 + 4 - 1$	2
-1	$y = -(-1)^2 + 4(-1) - 1$ $y = -1 - 4 - 1$	-6

Graph the Parabola

Plot the vertex and the points from your table of values: (2,3), (1,2), (-1,-6).

Use the symmetry of parabolas to plot two more points on the "other side" of the graph. The point (1,2) is one unit away from the line of symmetry, so we can also plot the point (3,2). The point (-1,-6) is three units away from the line of symmetry, so we can also plot the point (5,-6).

Sketch in the parabola.



You Try It

Find the vertex of the following quadratic equations. Make a table of values and graph the parabola.

1. $y = x^2 - 4x$
2. $y = -2x^2 + 3$
3. $y = x^2 - 6x + 4$

Problem 1

$$y = x^2 - 4x$$

$a = 1$ $b = -4$

$$x = -\frac{b}{2a}$$

$$x = -\frac{-4}{2(1)}$$

$$x = 2$$

$$y = x^2 - 4x$$

$$y = 2^2 - 4(2)$$

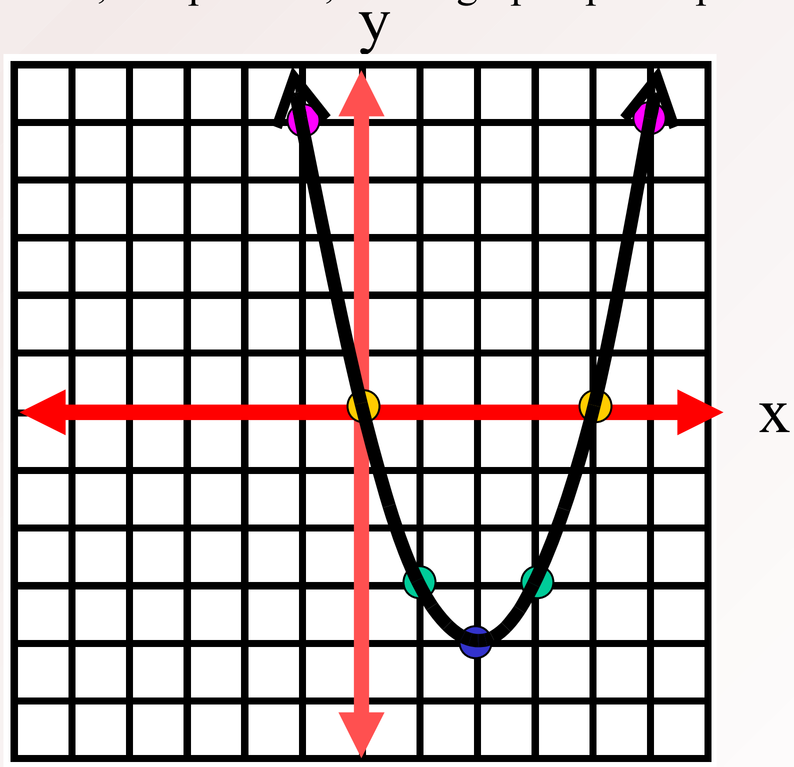
$$y = 4 - 8$$

$$y = -4$$

The vertex is at (2,-4)

x	$y = x^2 - 4x$	y
1	$y=1^2 - 4(1)$	-3
0	$y = 0^2 - 4(0)$	0

Notice, *a* is positive, so the graph opens up.



Problem 2

$$y = -2x^2 + 3$$

$a = -2 \quad b = 0$

$$x = -\frac{b}{2a}$$

$$x = -\frac{0}{2(-2)}$$

$$x = 0$$

$$y = -2x^2 + 3$$

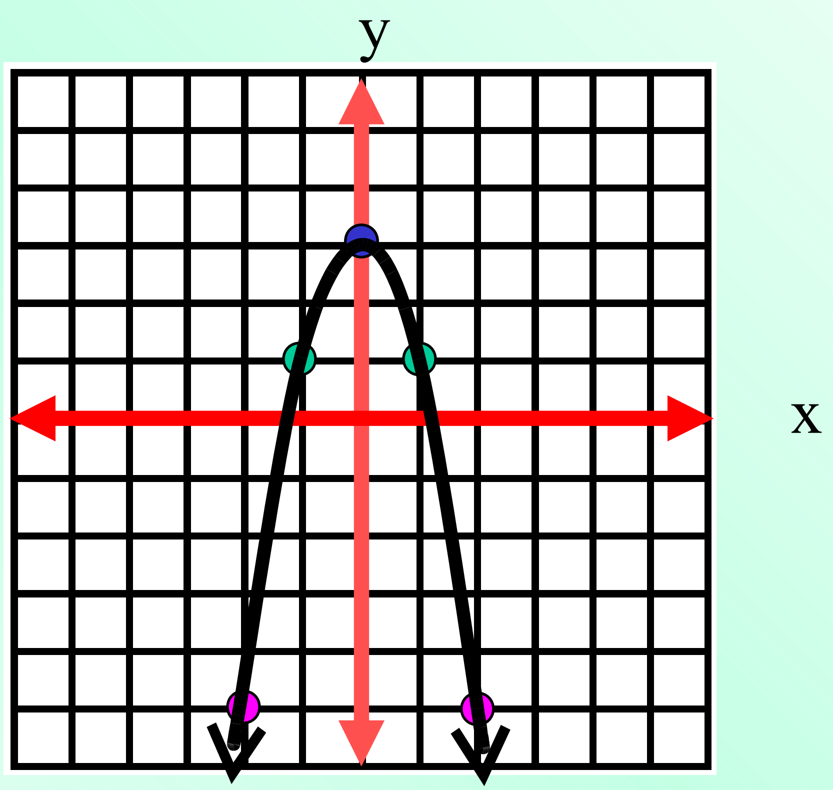
$$y = -2(0)^2 + 3$$

$$y = 3$$

The vertex is at (0,3)

x	$y = -2x^2 + 3$	y
-1	$y = -2(-1)^2 + 3$	1
-2	$y = -2(-2)^2 + 3$	-5

Notice, *a* is negative, so the graph opens down.



Problem 3

$$y = x^2 - 6x + 4$$

$a = 1 \quad b = -6$

$$x = -\frac{b}{2a}$$

$$x = -\frac{-6}{2(1)}$$

$$x = 3$$

$$y = x^2 - 6x + 4$$

$$y = 3^2 - 6(3) + 4$$

$$y = -5$$

The vertex is at (3,-5)

x	$y = x^2 - 6x + 4$	y
1	$y = 1^2 - 6(1) + 4$	-1
0	$y = 0^2 - 6(0) + 4$	4

Notice, *a* is positive, so the graph opens up.