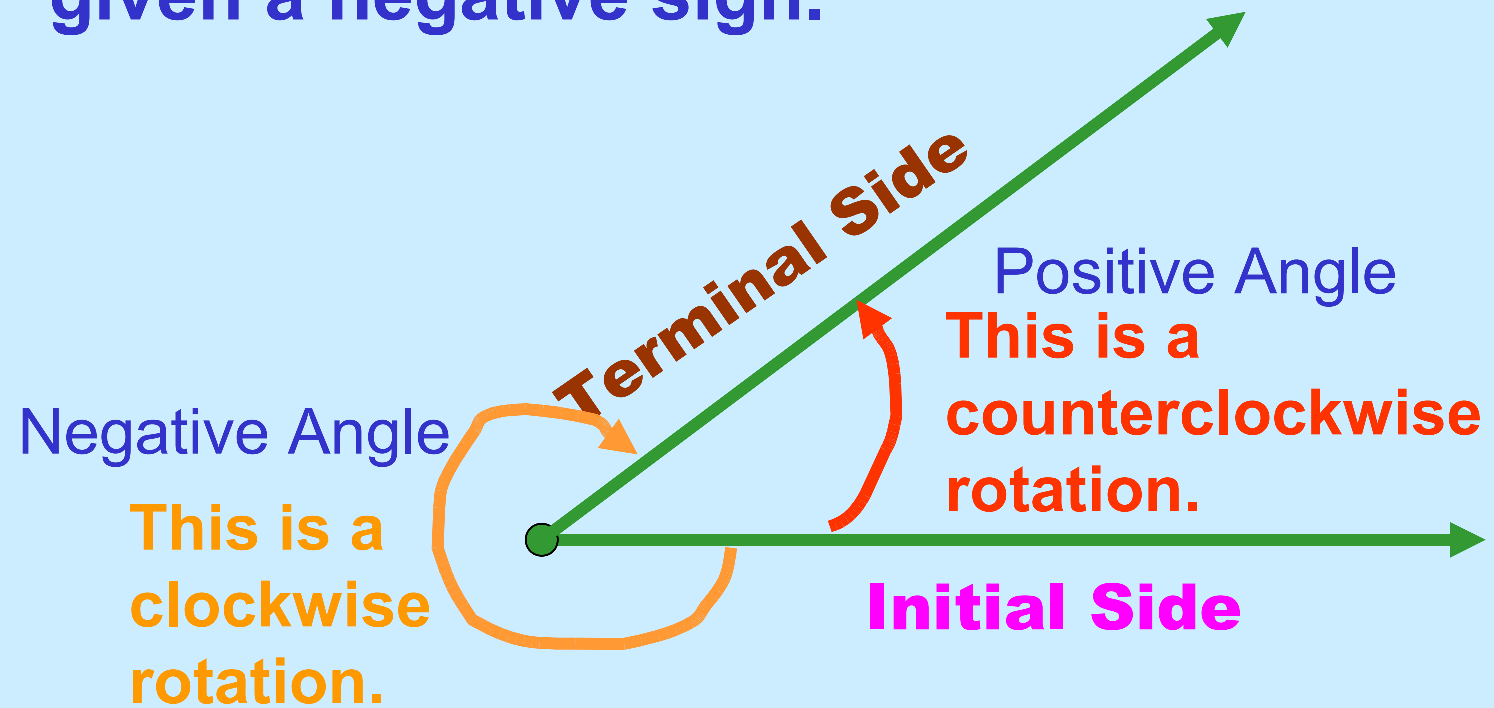
Angles and Their Measure

An angle is formed by joining the endpoints of two half-lines called rays.

The side you measure to is called the terminal side.

Angles measured counterclockwise are given a positive sign and angles measured clockwise are given a negative sign.



The side you measure from is called the initial side.

It's Greek To Me!

It is customary to use small letters in the Greek alphabet to symbolize angle measurement.

α

alpha

β

beta

γ

gamma

θ

theta

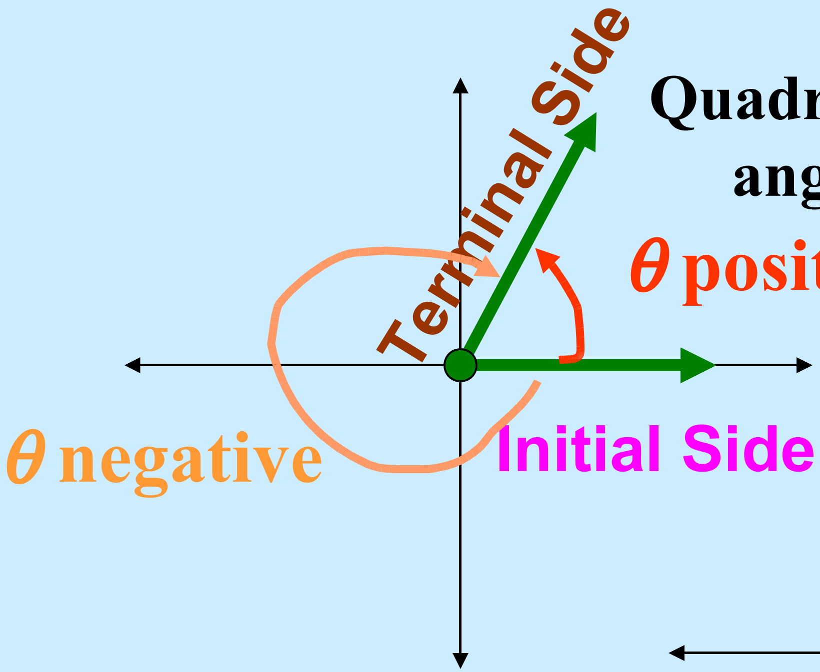
ϕ

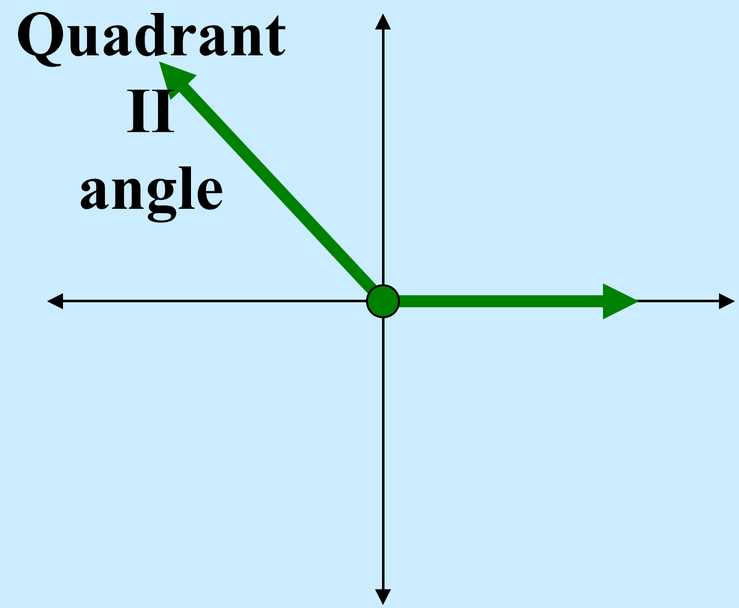
phi

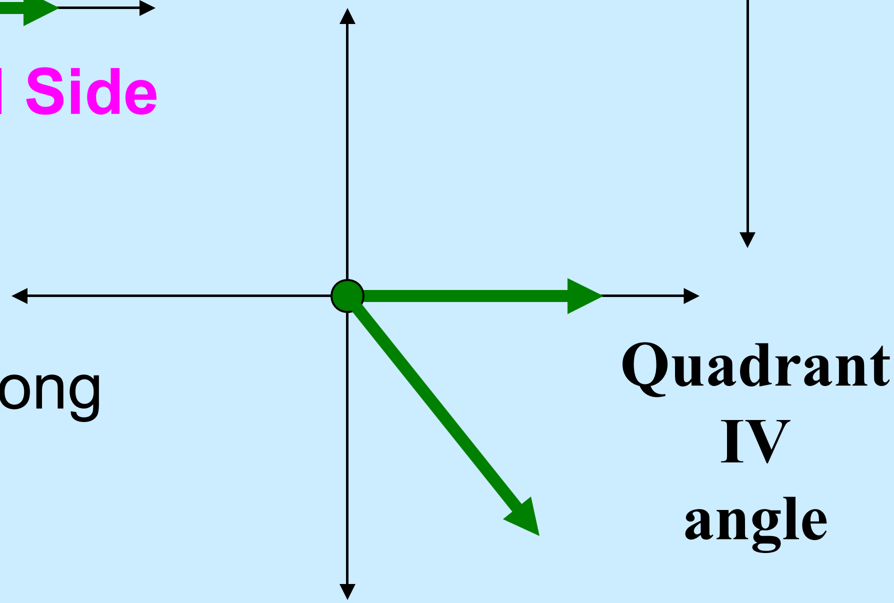
δ

delta

We can use a coordinate system with angles by putting the initial side along the positive *x*-axis with the vertex at the origin.



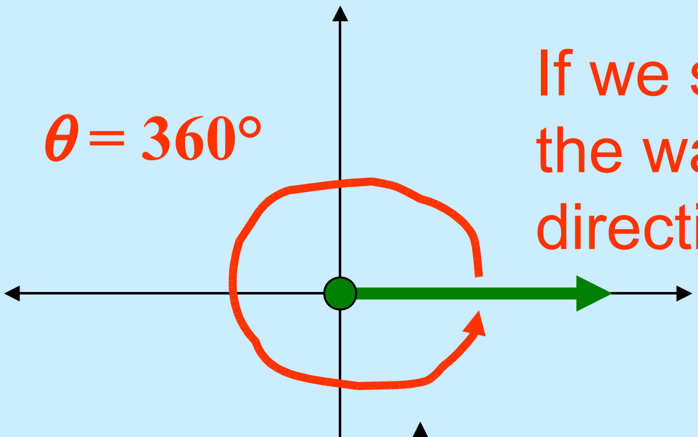




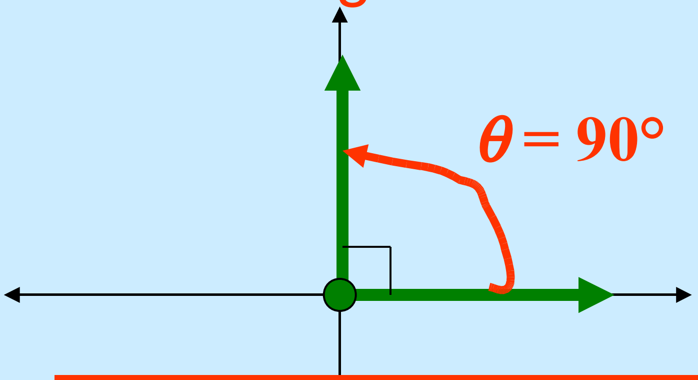
If the terminal side is along an axis it is called a quadrantal angle.

We say the angle lies in whatever quadrant the terminal side lies in.

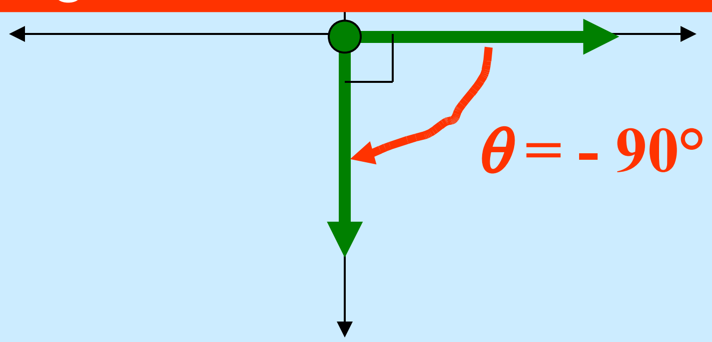
We will be using two different units of measure when talking about angles: Degrees and Radians



If we start with the initial side and go all of the way around in a counterclockwise direction we have 360 degrees



If we went 1/4 of the way in a clockwise direction the angle would measure -90°



You are probably already familiar with a right angle that measures 1/4 of the way around or 90°

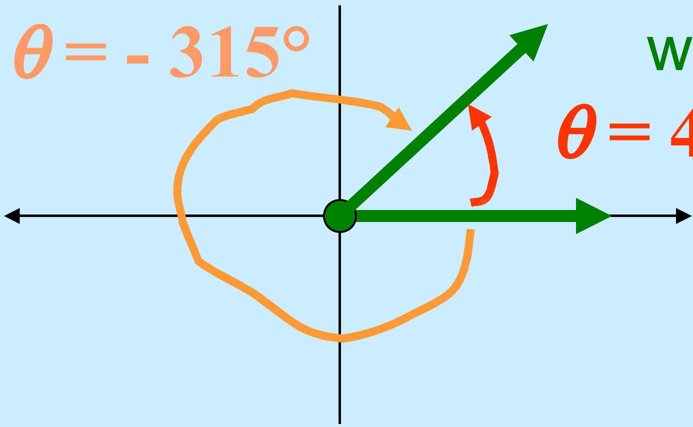
Let's talk about degrees first. You are probably already somewhat familiar with degrees.

What is the measure of this angle?

You could measure in the positive direction and go around another rotation which would be another 360°

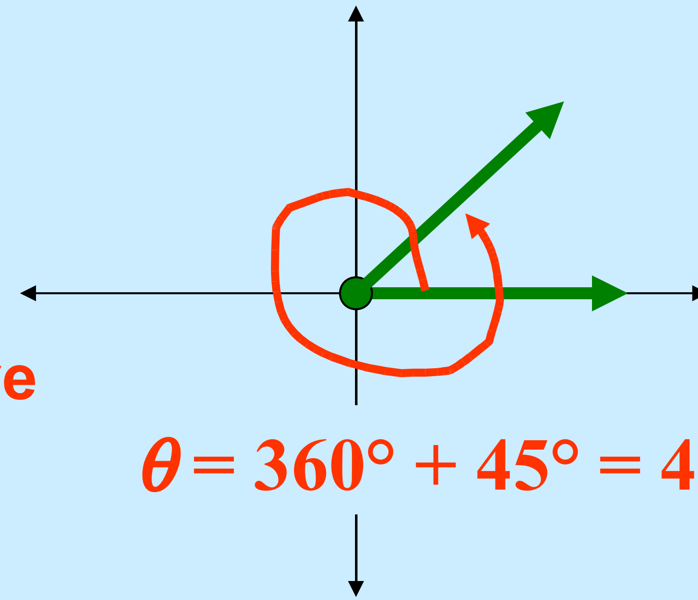
$\theta = -360° + 45°$

$\theta = -315°$



$\theta = 45°$

You could measure in the positive direction



$\theta = 360° + 45° = 405°$

You could measure in the negative direction

There are many ways to express the given angle. Whichever way you express it, it is still a Quadrant I angle since the terminal side is in Quadrant I.

If the angle is not exactly to the next degree it can be expressed as a decimal (most common in math) or in degrees, minutes and seconds (common in surveying and some navigation).

1 degree = 60 minutes   1 minute = 60 seconds

$$\theta = 25°48'30''$$

degrees   minutes   seconds

To convert to decimal form use conversion fractions. These are fractions where the numerator = denominator but two different units. Put unit on top you want to convert to and put unit on bottom you want to get rid of.

Let's convert the seconds to minutes

$$30'' \cdot \frac{1'}{60''} = 0.5'$$

1 degree = 60 minutes   1 minute = 60 seconds

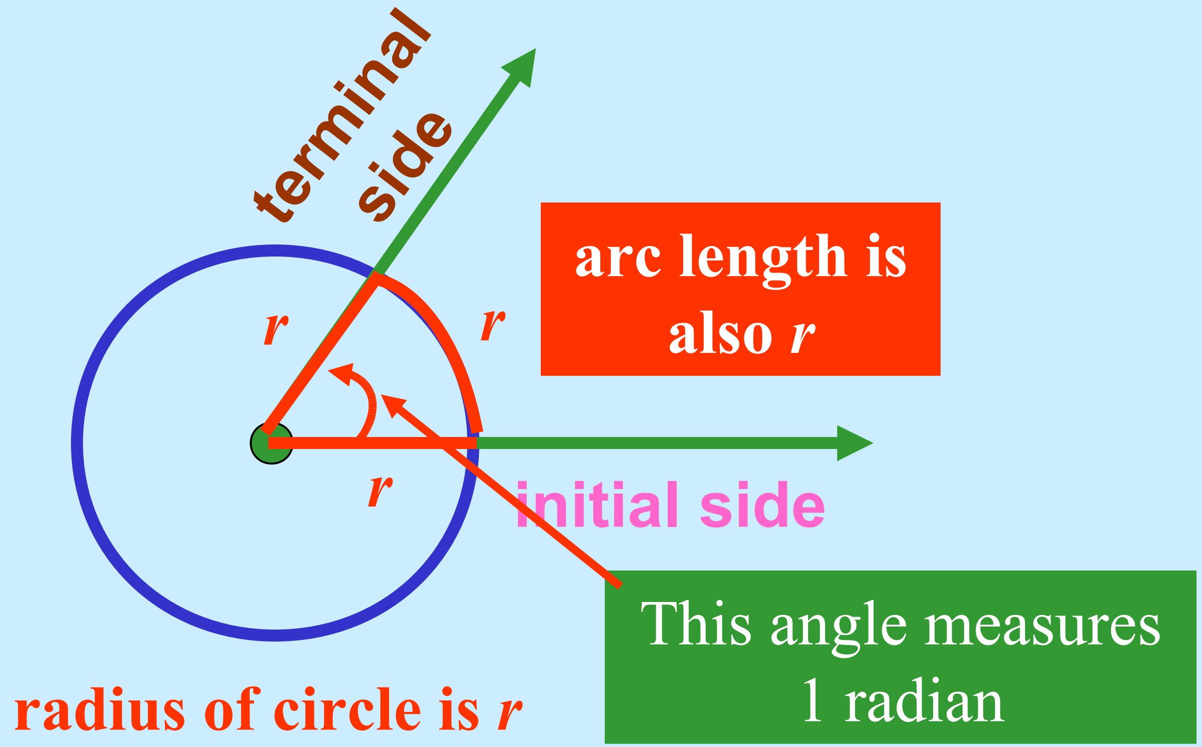
θ = 25°48'30" = 25°48.5' = 25.808°

Now let's use another conversion fraction to get rid of minutes.

$$48.5' \cdot \frac{1^\circ}{60'} = .808^\circ$$

Another way to measure angles is using what is called *radians*.

Given a circle of radius *r* with the vertex of an angle as the center of the circle, if the arc length formed by intercepting the circle with the sides of the angle is the same length as the radius *r*, the angle measures one radian.



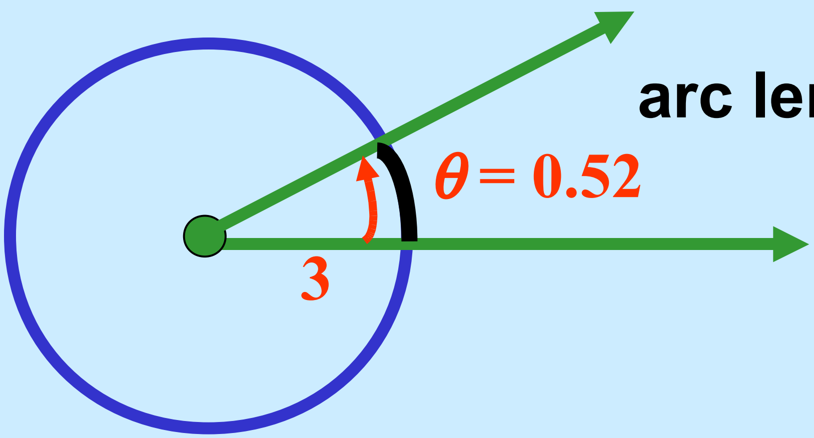
Arc length *s* of a circle is found with the following formula:

$$s = r\theta$$

IMPORTANT: ANGLE MEASURE MUST BE IN RADIANS TO USE FORMULA!

arc length | radius | measure of angle

Find the arc length if we have a circle with a radius of 3 meters and central angle of 0.52 radian.



arc length to find is in black

$$S = 3\ 0.52 = 1.56 \text{ m}$$

What if we have the measure of the angle in degrees? We can't use the formula until we convert to radians, but how?

We need a conversion from degrees to radians. We could use a conversion fraction if we knew how many degrees equaled how many radians.

Let's start with the arc length formula

$$s = r\theta$$

If we look at one revolution around the circle, the arc length would be the circumference. Recall that circumference of a circle is $2\pi r$

$$2\pi r = r\theta$$

cancel the *r*'s

$$2\pi = \theta$$

This tells us that the radian measure all the way around is 2π. All the way around in degrees is 360°.

2 π radians = 360°

2π radians = 360° **π radians = 180°**

Convert 30° to radians using a conversion fraction.

$$30^\circ \cdot \frac{2\pi \text{ radians}}{360^\circ}$$

180°

The fraction can be reduced by 2. This would be a simpler conversion fraction.

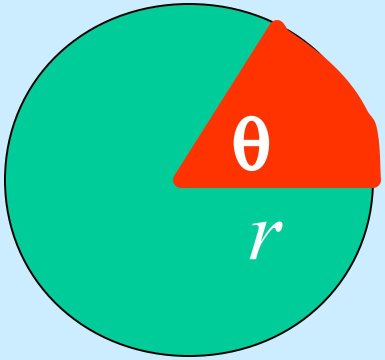
$$= \frac{\pi}{6} \textbf{ radians} = \textbf{0.52}$$

Can leave with π or use π button on your calculator for decimal.

Convert $\pi/3$ radians to degrees using a conversion fraction.

$$\frac{\pi}{3}\text{radians} \cdot \frac{180^\circ}{\pi \text{ radians}} = 60^\circ$$

Area of a Sector of a Circle



The formula for the area of a sector of a circle (shown in red here) is derived in your textbook. It is:

$$A = \frac{1}{2} r^2 \theta$$

Again θ must be in RADIANS so if it is in degrees you must convert to radians to use the formula.

Find the area of the sector if the radius is 3 feet and $\theta = 50°$

$$50° \cdot \frac{\pi \text{ radians}}{180°} = 0.873 \text{ radians}$$

$$A = \left(\frac{1}{2}\right)(3)^2(0.873)$$

$$= 3.93 \text{ sq ft}$$

Circular Motion

$$\omega = \frac{\theta}{t}$$

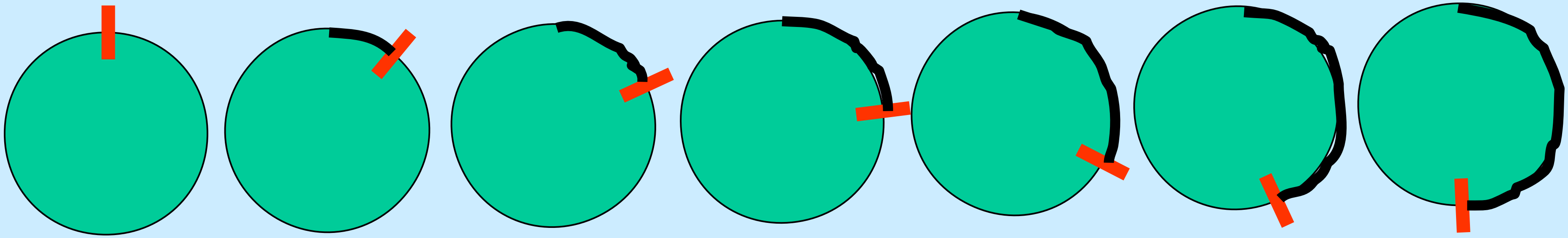
θ must be in radians

When something is rotating we can measure how fast the angle is changing called angular speed. This is generally designated by the Greek letter ω.

ω is the angular speed

t is time

We can also compute linear speed of an object that is rotating. Think of placing a mark on a wheel and then keeping track of how far the mark travels. It would travel around the circle so it makes sense to use arc length *s* for the distance.



The mark traveled the distance of $r\theta$ (since $s = r\theta$)

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{linear speed } v = \frac{s}{t}$$

$$\omega = \frac{\theta}{t}$$

So we have **angular speed ω** and **linear speed v**. We can get a another formula for linear speed by doing some substituting.

$$v = \frac{s}{t}$$

$$s = r\theta \qquad \theta = \omega t$$

This is angular speed formula but solved for θ

$$v = \frac{s}{t} \qquad v = \frac{r\theta}{t}$$

$$v = \frac{r\omega t}{t}$$

$$v = r\omega$$

$$\omega = \frac{\theta}{t} \qquad v = r\omega \qquad v = \frac{s}{t}$$

Let's try a problem: The radius of each wheel of a car is 15 inches. If the wheels are turning at the rate of 3 revolutions per second, how fast is the car moving?

We want linear speed so we'll use

$$v = r\omega$$

To find ω we'll use the angular speed formula

$$\omega = \frac{\theta}{t}$$

We need radians per time but we have revolutions per second so we'll convert that to radians per second.

conversion fraction since 1 revolution is 2π radians

$$\frac{3 \text{ rev}}{\text{sec}} \cdot \frac{2\pi \text{ radians}}{1 \text{ rev}} = \frac{6\pi \text{ radians}}{\text{sec}}$$

$$v = 15(6\pi)\frac{\text{in}}{\text{sec}}$$

$$v = 90\pi \frac{\text{in}}{\text{sec}}$$

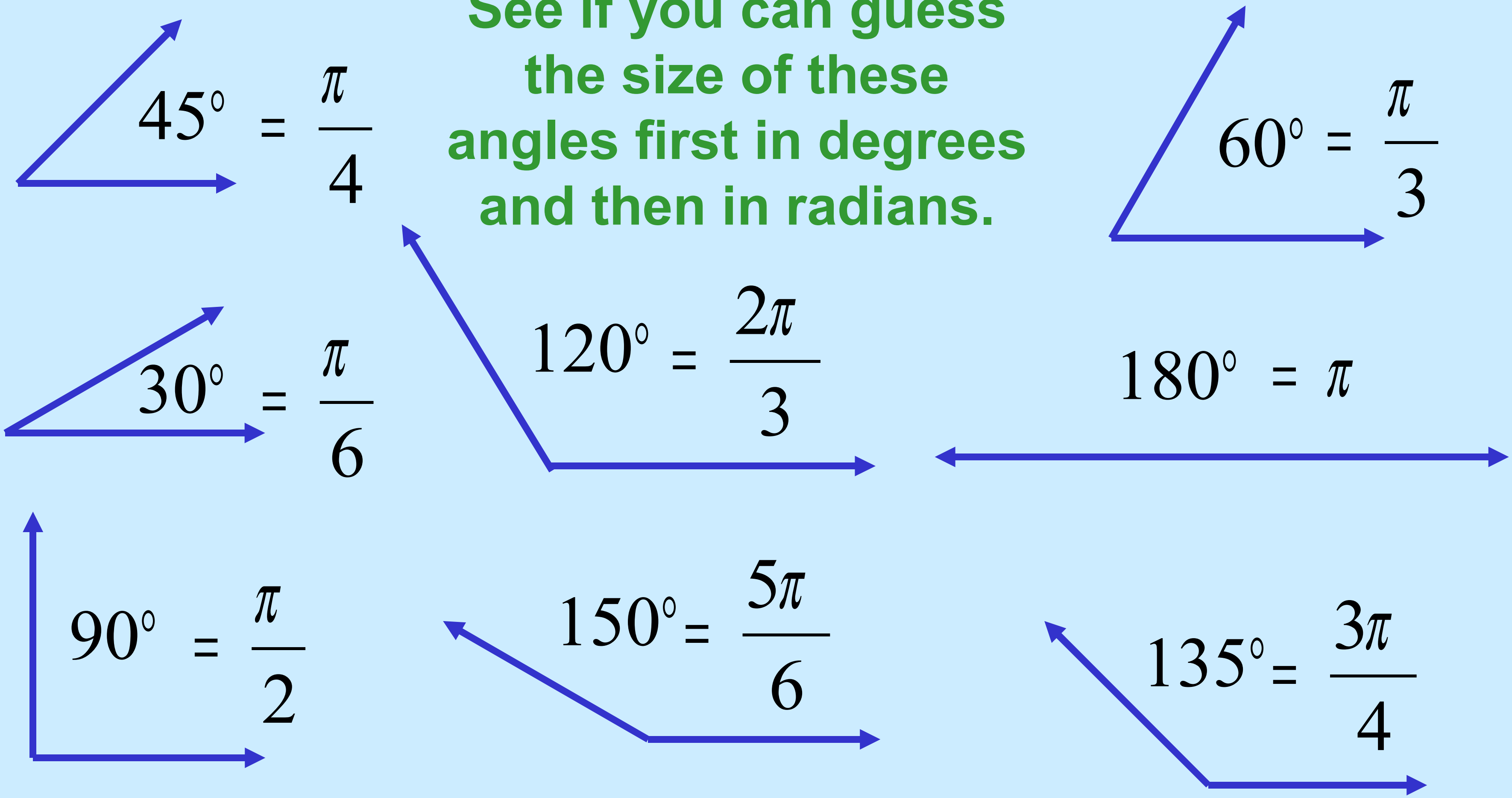
Since we don't have a feel for how fast inches per second is in a car, let's convert this to miles per hour using conversion fractions and the pi button on the calculator.

$$v = 282.7 \frac{\text{in}}{\text{sec}} \cdot \frac{1\text{ ft}}{12\text{ in}} \cdot \frac{1\text{ mile}}{5280\text{ feet}} \cdot \frac{60\text{ sec}}{1\text{ min}} \cdot \frac{60\text{ min}}{1\text{ hour}}$$

$$v = 16.1 \frac{\text{miles}}{\text{hour}}$$

A Sense of Angle Sizes

See if you can guess the size of these angles first in degrees and then in radians.



You will be working so much with these angles, you should know them in both degrees and radians.